Differential geometry for physicists - Assignment 15

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1. **Canonical flat connection**
   Consider the Cartesian product $P = M \times G$ of a manifold M and a Lie group G together with the right action $(x, g) \cdot h = (x, gh)$ and projection $\pi = \mathrm{pr}_M$ onto the first factor.
   (a) Show that $\pi : P \to M$ is a principal fiber bundle.
   (b) Show that the map
   $$\begin{array}{cccc} \omega & : & P & \to & J^1(P) \\ & & (x,g) & \mapsto & j^1_x\sigma_{(x,g)} \end{array},$$
   where the section $\sigma_{(x,g)}$ is given by
   $$\begin{array}{cccc} \sigma_{(x,g)} & : & M & \to & P \\ & & y & \mapsto & (y,g) \end{array},$$
   is a principal Ehresmann connection.
   (c) Calculate the connection form $\theta : TP \to VP$, horizontal distribution HP and principal G-connection $\vartheta \in \Omega^1(P, \mathfrak{g})$.

2. **Associated bundle connection**
   Let $\pi : P \to M$ be the principal bundle from the previous exercise, $F = \mathbb{R}^n$ and the left action $\rho : G \times F \to F$ be linear, i.e., $\rho(g, f) = \rho(g)f$, where $\rho(g) \in \mathrm{GL}(n)$.
   (a) Show that the associated bundle $P \times_\rho F$ is diffeomorphic to the Cartesian product $M \times F$.
   (b) Calculate the associated bundle connection $\omega_\rho : P \times_\rho F \to J^1(P \times_\rho F)$ and the corresponding Koszul connection ∇^{ω_ρ}.