Differential geometry for physicists - Assignment 13

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1. **Rotational symmetry**
   Consider the trivial fiber bundle $\pi : M \times Q \to M$ with $M = \mathbb{R}$ and $Q = \mathbb{R}^k$ and a Lagrangian
   $$L = \mathcal{L}(\delta_{ab}q^a q^b, \delta_{ab}\dot{q}^a\dot{q}^b)dt \in \Omega^{1,0}(J^\infty(E))\,,$$
   which depends only on the squared distance $\delta_{ab}q^a q^b$ from the origin of $\mathbb{R}^k$ and the square $\delta_{ab}\dot{q}^a\dot{q}^b$ of the velocity.

   (a) Calculate $d_V L$ and $\mathcal{E}L$.

   (b) Show that $\mathcal{E}L - d_V L = d_H\eta$ is d_H-exact and determine η.

   (c) Calculate the prolongation $\operatorname{pr} X$ of the evolutionary vector field $X = \omega^a{}_b q^b \bar{\partial}_a$, where ω is a constant, antisymmetric matrix, $\delta_{ab}\omega^b{}_c + \delta_{cb}\omega^b{}_a = 0$.

   (d) Show that X is a symmetry of the Lagrangian.

   (e) Calculate the conserved current ψ corresponding to X.

   (f) Use the Euler-Lagrange equations to check that ψ is indeed conserved.