Differential geometry for physicists - Assignment 9

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21. April 2015

1. **Contact forms**
   Let $M = \mathbb{R}$ with coordinate x, $E = \mathbb{R}^2$ with coordinates (x, y) and $\pi : E \to M$ the projection onto the first component. Denote the coordinates on the infinite jet space $J^\infty(E)$ by $(x, y_{(0)}, y_{(1)}, \ldots)$.

   (a) Write out the formula for the contact forms $\theta_{(n)}$ depending on n using the coordinates above.

   (b) Consider a section $\sigma : M \to E$ written in the form $x \mapsto y(x)$ and its ∞-jet prolongation $j^\infty\sigma$ written as $x \mapsto (y_{(0)}(x), y_{(1)}(x), \ldots)$. Write out a formula for the components $y_{(k)}(x)$.

   (c) Calculate the pullback of the contact forms $\theta_{(n)}$ to M along $j^\infty\sigma$ given above. What do you see?

2. **Differential operators**
   Consider the same bundle as above and let d_H and d_V be the horizontal and the vertical derivatives.

   (a) Let $\omega(x, y_{(0)}, y_{(1)}, \ldots) = f(x, y_{(0)}, y_{(1)})dx \in \Omega^{1,0}(J^\infty(E))$, i.e., f should depend only on the coordinates $x, y_{(0)}, y_{(1)}$. Calculate $d_H\omega$ and $d_V\omega$.

   (b) Consider a function $g(x, y_{(0)}, y_{(1)}) \in \Omega^{0,0}(J^\infty(E))$. Calculate $d_H g$ and its pullback $(j^\infty\sigma)^*(d_H g)$ to M along the ∞-jet prolongation $j^\infty\sigma$ from the previous exercise. Show that there exists a function $\tilde{g} \in C^\infty(M, \mathbb{R})$ such that $(j^\infty\sigma)^*(d_H g) = d\tilde{g}$ and calculate $\tilde{g}$.