Differential geometry for physicists - Assignment 8

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1. **Coordinate transformation on jet manifolds**
   Let $M = \mathbb{R}$ with coordinate x and $N = \mathbb{R}$ with coordinate y.

   (a) Write out explicitly the coordinates y_Λ for all relevant Λ derived from x and y on $J_0^2(M, N)$. What is the dimension of this manifold?

   (b) Let $h \in C^\infty(M, N)$ be a smooth map, given in coordinates as $y(x)$. What are the coordinates of $j_0^2 h$?

   (c) Let $\tilde{x} = f(x)$ and $\tilde{y} = g(y)$ new coordinates on M, which introduce new coordinates $\tilde{y}_{\tilde{\Lambda}}$ on $J_0^2(M, N)$. Rewrite h using the new coordinates $\tilde{x}$ and $\tilde{y}$. What are the new coordinates of $j_0^2 h$?

   (d) Rewrite your result as a formula for the coordinates y_Λ on $J_0^2(M, N)$ as functions of the coordinates $\tilde{y}_{\tilde{\Lambda}}$.

   Hint: You should find the derivatives $dy/dx(0), d^2y/dx^2(0)$ of the coordinate expression $y(x)$ of h somewhere, and the same for its coordinate expression $\tilde{y}(\tilde{x})$. How are they related?

2. **Dimension of jet bundles**
   Let $\pi : T_s^r M \to M$ be the tensor bundle of type (r, s) over a manifold M of dimension $\dim M = n$. What is the dimension of the k'th order jet bundle $J^k(T_s^r M)$?

3. **Classical mechanics?**
   Let $M = \mathbb{R}$ and Q a manifold of dimension n. Let $E = \mathbb{R} \times Q$ be the trivial fiber bundle with projection $\pi : \mathbb{R} \times Q \to \mathbb{R}$ onto the first factor.

   (a) Introduce suitable coordinates on $\mathbb{R}$, Q and $\mathbb{R} \times Q$.

   (b) Introduce coordinates on the first-order jet bundle $J^1(E)$.

   (c) Consider a function $\mathcal{L} \in C^\infty(J^1(E), \mathbb{R})$. Explain what this function looks like in the context of classical mechanics, and which objects in the construction above correspond to "time" and "space".