Differential geometry for physicists - Assignment 7

Manuel Hohmann

7. April 2015

1. **Integral over the sphere**
   Consider a chart of the sphere S^2 corresponding to the usual latitude / longitude coordinates $\theta \in (-\pi/2, \pi/2)$ and $\phi \in (0, 2\pi)$. Use this chart to define the 2-form $\omega = \sin\theta \, d\theta \wedge d\phi$.

   (a) Show that ω is indeed a 2-form, i.e., a smooth section of $\Lambda^2 T^*M$. You may use the two charts given by stereographic projection we defined earlier for this purpose.

   (b) Calculate the integral of ω over the 2-cube

$$\begin{array}{rcccc} c & : & [0,1]^2 & \to & S^2 \\ & & (x,y) & \mapsto & \left(\pi\left(x - \frac{1}{2}\right), 2\pi y\right) \end{array} .$$

2. **Stokes in $\mathbb{R}^3$**
   Let $M = \mathbb{R}^3$ and $\omega = x \, dy \wedge dz \in \Omega^2(M)$. Consider the 3-cube

$$\begin{array}{rcccc} c & : & [0,1]^3 & \to & M \\ & & (x,y,z) & \mapsto & (x,y,z) \end{array} .$$

   (a) Determine the 2-chain ∂c (the boundary of c).

   (b) Calculate $d\omega$.

   (c) Calculate the integrals

$$\int_{\partial c} \omega \quad \text{and} \quad \int_c d\omega \, .$$