Differential geometry for physicists - Assignment 2

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1. **Hairy ball**
   Imagine you have a sphere S^2 and a flag which you can place at some point on the sphere, such that it points into some direction tangent to the sphere. Let M denote the configuration space of this flag, i.e., the space containing all possible ways to place the flag on the sphere. Each element of this space determines the position and direction of the flag. Let $\pi : M \to S^2$ the function that "forgets" about the orientation of the flag and assigns to a flag's configuration its position on the sphere.

   (a) Use the charts of the sphere to construct charts of M. (Hint: You might wish to construct two charts for M for each chart of S^2.)

   (b) Show that $\pi : M \to S^2$ is a smooth surjective map.

   (c) Show that (M, S^2, π, S^1) is a fiber bundle (which is called the *unit tangent bundle* of S^2).

2. **Infinite Möbius strip**
   Let $(M, S^1, \pi, \mathbb{R})$ be the infinite Möbius strip as defined in the lecture.

   (a) Show that $(M, S^1, \pi, \mathbb{R})$ is a vector bundle of rank 1.

   (b) Let $f : S^1 \to M$ and $g : S^1 \to M$ be sections. Show that there is at least one $p \in S^1$ such that $f(p) = g(p)$.

3. **Zero section**
   Show that the zero section of a vector bundle is indeed a section.