Differential geometry for physicists - Assignment 1

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10. February 2015

1. **Know your coordinates**

   (a) Your best friend tells you: "Look! I take the sphere S^2 with coordinates given by latitude $-\pi/2 \leq \theta \leq \pi/2$ and longitude $0 \leq \varphi < 2\pi$ and define the map $f : S^2 \to \mathbb{R}$ as $f(\theta, \varphi) = \sin\varphi$. This is a smooth map from S^2 to $\mathbb{R}$." Explain why he is doing something terribly wrong, and why f is not even a function on the sphere at all.

   (b) Your best friend listens to your argument and continues: "But our physics teacher did just the same with the function $g(\theta, \varphi) = \cos\theta \sin\varphi$ and he said it's a smooth map!" Explain why the physics teacher is correct, so that g is indeed a smooth map.

   (c) Write the map g using the two charts of the sphere given in the lecture.

2. **Product manifold**
   Show that $\mathcal{A}_{M\times N}$ in the definition of a product manifold is an atlas and that the projections pr_M and pr_N are smooth maps.

3. **Map composition**
   Let L, M, N be manifolds and $f : L \to M$ and $g : M \to N$ smooth maps.

   (a) Show that the composition $g \circ f$ is a smooth map from L to N.

   (b) If both f and g are diffeomorphisms, is $g \circ f$ also a diffeomorphism? Explain your answer.