Disformal transformations in scalar-torsion gravity

arXiv:1905.00451 [gr-qc]

Manuel Hohmann

Laboratory of Theoretical Physics - Institute of Physics - University of Tartu
Center of Excellence “The Dark Side of the Universe”

European Union
European Regional
Development Fund

Investing
in your future

Geometric Foundations of Gravity - 17. 6. 2019

Outline

1. Introduction
2. Teleparallel disformal transformations
3. Construction of an invariant action
4. The quadratic class of actions
5. Conclusion

Outline

1. Introduction
2. Teleparallel disformal transformations
3. Construction of an invariant action
4. The quadratic class of actions
5. Conclusion

Why study disformal transformations in scalar-torsion gravity?

- Brief overview of scalar-torsion theories of gravity:
  - Teleparallel dark energy [Geng '11]
  - Conformally coupled scalar fields [Maluf, Faria '11] [Bamba, Odintsov, Saez-Gomez '13] [Wright '16]
  - Coupling to teleparallel boundary term [Bahamonde, Wright '15] [Bahamonde, Marciu, Rudra '18]
  - Covariant formulation of scalar-torsion gravity [MH, Järv, Ualikhanova '18]
  - Most general scalar-torsion gravity and conformal transformations [MH '18]
  - Non-minimally coupled $L(T, X, Y, \phi)$ class of theories [MH, Pfeifer '18]
  - Analogue and extension of classical scalar-curvature gravity [MH '18]
  - Teleparallel Horndeski gravity [Bahamonde, Dialektopoulos, Said '19]

Why study disformal transformations in scalar-torsion gravity?

- Brief overview of scalar-torsion theories of gravity:
  - Teleparallel dark energy [Geng '11]
  - Conformally coupled scalar fields [Maluf, Faria '11] [Bamba, Odintsov, Saez-Gomez '13] [Wright '16]
  - Coupling to teleparallel boundary term [Bahamonde, Wright '15] [Bahamonde, Marciu, Rudra '18]
  - Covariant formulation of scalar-torsion gravity [MH, Järv, Ualikhanova '18]
  - Most general scalar-torsion gravity and conformal transformations [MH '18]
  - Non-minimally coupled $L(T, X, Y, \phi)$ class of theories [MH, Pfeifer '18]
  - Analogue and extension of classical scalar-curvature gravity [MH '18]
  - Teleparallel Horndeski gravity [Bahamonde, Dialektopoulos, Said '19]
- Lessons to learn from scalar-curvature and Horndeski gravity theories:
  - Conformal frame freedom in scalar-curvature gravity [Flanagan '04]
  - Invariant formulation of scalar-curvature gravity [Järv, Kuusk, Saal, Vilson '14]
  - Invariance of Horndeski gravity under special disformal transformations [Bettoni, Liberati '13]
  - Disformal transformations and beyond Horndeski theories [Zumalacarregui, Garcia-Bellido '13]
  - Disformal and extended disformal transformations [Ezquiaga, Garcia-Bellido, Zumalacarregui '17]

Why study disformal transformations in scalar-torsion gravity?

- Brief overview of scalar-torsion theories of gravity:
  - Teleparallel dark energy [Geng '11]
  - Conformally coupled scalar fields [Maluf, Faria '11] [Bamba, Odintsov, Saez-Gomez '13] [Wright '16]
  - Coupling to teleparallel boundary term [Bahamonde, Wright '15] [Bahamonde, Marciu, Rudra '18]
  - Covariant formulation of scalar-torsion gravity [MH, Järv, Ualikhanova '18]
  - Most general scalar-torsion gravity and conformal transformations [MH '18]
  - Non-minimally coupled $L(T, X, Y, \phi)$ class of theories [MH, Pfeifer '18]
  - Analogue and extension of classical scalar-curvature gravity [MH '18]
  - Teleparallel Horndeski gravity [Bahamonde, Dialektopoulos, Said '19]
- Lessons to learn from scalar-curvature and Horndeski gravity theories:
  - Conformal frame freedom in scalar-curvature gravity [Flanagan '04]
  - Invariant formulation of scalar-curvature gravity [Järv, Kuusk, Saal, Vilson '14]
  - Invariance of Horndeski gravity under special disformal transformations [Bettoni, Liberati '13]
  - Disformal transformations and beyond Horndeski theories [Zumalacarregui, Garcia-Bellido '13]
  - Disformal and extended disformal transformations [Ezquiaga, Garcia-Bellido, Zumalacarregui '17]
- Arising questions:
  - Classes of scalar-torsion gravity with disformal invariance?
  - Disformal invariance of teleparallel Horndeski gravity?
  - Healthy "teleparallel beyond Horndeski" theories via disformal transformations?
  - Invariant formulation in terms of disformal invariants?

Ingredients of scalar-torsion gravity

- Fundamental fields:
  - Coframe field $\theta^a = \theta^a{}_\mu dx^\mu$.
  - Flat spin connection $\omega^a{}_b = \omega^a{}_{b\mu} dx^\mu$.
  - Scalar field ϕ.
  - Arbitrary matter fields χ^I.

Ingredients of scalar-torsion gravity

- Fundamental fields:
  - Coframe field $\theta^a = \theta^a{}_\mu \mathrm{d}x^\mu$.
  - Flat spin connection $\omega^a{}_b = \omega^a{}_{b\mu} \mathrm{d}x^\mu$.
  - Scalar field ϕ.
  - Arbitrary matter fields χ^I.
- Derived quantities:
  - Frame field $e_a = e_a{}^\mu \partial_\mu$ with $e_a \lrcorner \theta^b = \delta_a^b$.
  - Metric $g_{\mu\nu} = \eta_{ab} \theta^a{}_\mu \theta^b{}_\nu$.
  - Volume form $\mathrm{vol}_\theta = \frac{1}{4!} \epsilon_{abcd} \theta^a \wedge \theta^b \wedge \theta^c \wedge \theta^d = \theta^0 \wedge \theta^1 \wedge \theta^2 \wedge \theta^3$.
  - Torsion $T^a = \mathrm{D}\theta^a = \mathrm{d}\theta^a + \omega^a{}_b \wedge \theta^b$.

Ingredients of scalar-torsion gravity

- Fundamental fields:
  - Coframe field $\theta^a = \theta^a{}_\mu \mathrm{d}x^\mu$.
  - Flat spin connection $\omega^a{}_b = \omega^a{}_{b\mu} \mathrm{d}x^\mu$.
  - Scalar field ϕ.
  - Arbitrary matter fields χ^I.
- Derived quantities:
  - Frame field $e_a = e_a{}^\mu \partial_\mu$ with $e_a \lrcorner \theta^b = \delta^b_a$.
  - Metric $g_{\mu\nu} = \eta_{ab} \theta^a{}_\mu \theta^b{}_\nu$.
  - Volume form $\mathrm{vol}_\theta = \frac{1}{4!} \epsilon_{abcd} \theta^a \wedge \theta^b \wedge \theta^c \wedge \theta^d = \theta^0 \wedge \theta^1 \wedge \theta^2 \wedge \theta^3$.
  - Torsion $T^a = \mathrm{D}\theta^a = \mathrm{d}\theta^a + \omega^a{}_b \wedge \theta^b$.
- Remarks on notations and conventions:
  - Interior product between vector field v and differential form σ: $v \lrcorner \sigma$.
  - Exterior covariant derivative: D.
  - Lorentz indices are raised and lowered with the Minkowski metric $\eta_{ab} = \mathrm{diag}(-1, 1, 1, 1)$.
  - "Musical" isomorphisms for vector field v and one-form σ:

$$\sigma^\sharp = (e_a \lrcorner \sigma) e^a = \eta^{ab} e_a{}^\nu \sigma_\nu e_b^\mu \partial_\mu = g^{\mu\nu} \sigma_\nu \partial_\mu \,,$$

$$v^\flat = (v \lrcorner \theta_a) \theta^a = \eta_{ab} v^\nu \theta^a{}_\nu \theta^b{}_\mu \mathrm{d}x^\mu = g_{\mu\nu} v^\nu \mathrm{d}x^\mu \,.$$

Differential form language for scalar fields

- Lie derivative with respect to frame vector fields:

$$\phi_{,a} = \mathcal{L}_{e_a}\phi = e_a \lrcorner \mathrm{d}\phi = e_a{}^{\mu}\partial_\mu\phi \quad \Rightarrow \quad \mathrm{d}\phi = \phi_{,a}\theta^a \,.$$

Differential form language for scalar fields

- Lie derivative with respect to frame vector fields:

$$\phi_{,a} = \mathcal{L}_{e_a}\phi = e_a \lrcorner \mathrm{d}\phi = e_a{}^{\mu}\partial_{\mu}\phi \quad \Rightarrow \quad \mathrm{d}\phi = \phi_{,a}\theta^a\,.$$

- First and second scalar field derivative one-forms:

$$\psi_a = \phi_{,a}\mathrm{d}\phi \quad \text{and} \quad \pi_a = \mathrm{D}\phi_{,a} = \mathrm{d}\phi_{,a} - \omega^b{}_a \wedge \phi_{,b}\,.$$

Differential form language for scalar fields

- Lie derivative with respect to frame vector fields:

$$\phi_{,a} = \mathcal{L}_{e_a}\phi = e_a \lrcorner \mathrm{d}\phi = e_a{}^{\mu}\partial_\mu\phi \quad \Rightarrow \quad \mathrm{d}\phi = \phi_{,a}\theta^a \,.$$

- First and second scalar field derivative one-forms:

$$\psi_a = \phi_{,a}\mathrm{d}\phi \quad \text{and} \quad \pi_a = \mathrm{D}\phi_{,a} = \mathrm{d}\phi_{,a} - \omega^b{}_a \wedge \phi_{,b} \,.$$

- Scalar field kinetic energy scalar:

$$X = -\frac{1}{2}e_a \lrcorner \psi^a = -\frac{1}{2}\eta^{ab}\phi_{,a}\phi_{,b} \,.$$

Helpful formulas for scalar field terms

- Wedge products:

$$\psi_a \wedge \psi_b = 0\,, \quad \psi_a \wedge \mathrm{d}\phi = 0\,, \quad \psi_a \wedge \theta^a = 0\,.$$

Helpful formulas for scalar field terms

- Wedge products:

$$\psi_a \wedge \psi_b = 0\,, \quad \psi_a \wedge \mathrm{d}\phi = 0\,, \quad \psi_a \wedge \theta^a = 0\,.$$

- Exterior covariant derivatives:

$$\mathrm{D}\psi_a = \pi_a \wedge \mathrm{d}\phi \quad \text{and} \quad \mathrm{D}\pi_a = 0\,.$$

Helpful formulas for scalar field terms

- Wedge products:

$$\psi_a \wedge \psi_b = 0 \,, \quad \psi_a \wedge \mathrm{d}\phi = 0 \,, \quad \psi_a \wedge \theta^a = 0 \,.$$

- Exterior covariant derivatives:

$$\mathrm{D}\psi_a = \pi_a \wedge \mathrm{d}\phi \quad \text{and} \quad \mathrm{D}\pi_a = 0 \,.$$

- Exterior derivative of kinetic energy scalar:

$$\mathrm{d}X = \mathrm{D}X = -\eta^{ab}\phi_{,a}\mathrm{D}\phi_{,b} = -\phi_{,a}\pi^a \,.$$

Outline

1. Introduction
2. Teleparallel disformal transformations
3. Construction of an invariant action
4. The quadratic class of actions
5. Conclusion

Definition and transformation of fundamental fields

- Disformal transformation defined by functions $\mathfrak{C}, \mathfrak{D}$ of scalar field and kinetic term:

$$\bar{\theta}^a = \mathfrak{C}(\phi, X)\theta^a + \mathfrak{D}(\phi, X)\psi^a \,.$$

Definition and transformation of fundamental fields

- Disformal transformation defined by functions $\mathfrak{C}, \mathfrak{D}$ of scalar field and kinetic term:

$$\bar{\theta}^a = \mathfrak{C}(\phi, X)\theta^a + \mathfrak{D}(\phi, X)\psi^a \,.$$

- Transformation of the spin connection:
  - Should preserve flatness and antisymmetry of the spin connection.
  - $\Rightarrow$ Can always be absorbed into local Lorentz transformation.
  - $\Rightarrow$ Keep spin connection unchanged: $\bar{\omega}^a{}_b = \omega^a{}_b$.

Definition and transformation of fundamental fields

- Disformal transformation defined by functions $\mathfrak{C}, \mathfrak{D}$ of scalar field and kinetic term:

$$\bar{\theta}^a = \mathfrak{C}(\phi, X)\theta^a + \mathfrak{D}(\phi, X)\psi^a \,.$$

- Transformation of the spin connection:
  - Should preserve flatness and antisymmetry of the spin connection.

  $\Rightarrow$ Can always be absorbed into local Lorentz transformation.

  $\Rightarrow$ Keep spin connection unchanged: $\bar{\omega}^a{}_b = \omega^a{}_b$.
- Bonus: scalar field redefinition:

$$\bar{\phi} = f(\phi) \,.$$

Definition and transformation of fundamental fields

- Disformal transformation defined by functions $\mathfrak{C}, \mathfrak{D}$ of scalar field and kinetic term:

$$\bar{\theta}^a = \mathfrak{C}(\phi, X)\theta^a + \mathfrak{D}(\phi, X)\psi^a \,.$$

- Transformation of the spin connection:
  - Should preserve flatness and antisymmetry of the spin connection.
  - ⇒ Can always be absorbed into local Lorentz transformation.
  - ⇒ Keep spin connection unchanged: $\bar{\omega}^a{}_b = \omega^a{}_b$.
- Bonus: scalar field redefinition:

$$\bar{\phi} = f(\phi) \,.$$

- Matter fields are kept unchanged: $\bar{\chi}^I = \chi^I$.

- Frame vector fields:

$$\bar{e}_a = \frac{1}{\mathfrak{C}}\left(e_a - \frac{\mathfrak{D}}{\mathfrak{C} - 2X\mathfrak{D}}\psi^\sharp_a\right) = \frac{1}{\mathfrak{C}}\left(\delta^b_a - \frac{\mathfrak{D}}{\mathfrak{C} - 2X\mathfrak{D}}\phi_{,a}\phi_{,c}\eta^{bc}\right)e_b\,.$$

Transformation of teleparallel geometry

- Frame vector fields:

$$\bar{e}_a = \frac{1}{\mathfrak{C}}\left(e_a - \frac{\mathfrak{D}}{\mathfrak{C} - 2X\mathfrak{D}}\psi^{\sharp}_a\right) = \frac{1}{\mathfrak{C}}\left(\delta^b_a - \frac{\mathfrak{D}}{\mathfrak{C} - 2X\mathfrak{D}}\phi_{,a}\phi_{,c}\eta^{bc}\right)e_b\,.$$

- Metric:

$$\bar{g} = \mathfrak{C}^2 g + 2\mathfrak{D}(\mathfrak{C} - X\mathfrak{D})\mathrm{d}\phi \otimes \mathrm{d}\phi\,.$$

Transformation of teleparallel geometry

- Frame vector fields:

$$\bar{e}_a = \frac{1}{\mathfrak{C}}\left(e_a - \frac{\mathfrak{D}}{\mathfrak{C} - 2X\mathfrak{D}}\psi^\sharp_a\right) = \frac{1}{\mathfrak{C}}\left(\delta^b_a - \frac{\mathfrak{D}}{\mathfrak{C} - 2X\mathfrak{D}}\phi_{,a}\phi_{,c}\eta^{bc}\right)e_b\,.$$

- Metric:

$$\bar{g} = \mathfrak{C}^2 g + 2\mathfrak{D}(\mathfrak{C} - X\mathfrak{D})\mathrm{d}\phi \otimes \mathrm{d}\phi\,.$$

- Volume form:

$$\mathrm{vol}_{\bar{\theta}} = \mathfrak{C}^3(\mathfrak{C} - 2X\mathfrak{D})\mathrm{vol}_\theta\,.$$

- Frame vector fields:

$$\bar{e}_a = \frac{1}{\mathfrak{C}}\left(e_a - \frac{\mathfrak{D}}{\mathfrak{C} - 2X\mathfrak{D}}\psi^\sharp_a\right) = \frac{1}{\mathfrak{C}}\left(\delta^b_a - \frac{\mathfrak{D}}{\mathfrak{C} - 2X\mathfrak{D}}\phi_{,a}\phi_{,c}\eta^{bc}\right)e_b\,.$$

- Metric:

$$\bar{g} = \mathfrak{C}^2 g + 2\mathfrak{D}(\mathfrak{C} - X\mathfrak{D})\mathrm{d}\phi \otimes \mathrm{d}\phi\,.$$

- Volume form:

$$\mathrm{vol}_{\bar{\theta}} = \mathfrak{C}^3(\mathfrak{C} - 2X\mathfrak{D})\mathrm{vol}_\theta\,.$$

- Torsion:

$$\bar{T}^a = \mathfrak{C}T^a + \mathfrak{C}_{,\phi}\mathrm{d}\phi \wedge \theta^a + \mathfrak{C}_{,X}\mathrm{d}X \wedge \theta^a + \mathfrak{D}\pi^a \wedge \mathrm{d}\phi + \mathfrak{D}_{,X}\mathrm{d}X \wedge \psi^a\,.$$

Transformation of scalar field terms

- Lie derivative with respect to frame vector fields:

$$\bar{\phi}_{,a} = \frac{f'}{\mathfrak{C} - 2X\mathfrak{D}}\phi_{,a}\,.$$

Transformation of scalar field terms

- Lie derivative with respect to frame vector fields:

$$\bar{\phi}_{,a} = \frac{f'}{\mathfrak{C} - 2X\mathfrak{D}}\phi_{,a}\,.$$

- Scalar field kinetic energy scalar:

$$\bar{X} = \left(\frac{f'}{\mathfrak{C} - 2X\mathfrak{D}}\right)^2 X\,.$$

Transformation of scalar field terms

- Lie derivative with respect to frame vector fields:

$$\bar{\phi}_{,a} = \frac{f'}{\mathfrak{C} - 2X\mathfrak{D}}\phi_{,a}\,.$$

- Scalar field kinetic energy scalar:

$$\bar{X} = \left(\frac{f'}{\mathfrak{C} - 2X\mathfrak{D}}\right)^2 X\,.$$

- First scalar field derivative one-form:

$$\bar{\psi}_a = \frac{f'^2}{\mathfrak{C} - 2X\mathfrak{D}}\psi_a\,.$$

Transformation of scalar field terms

- Lie derivative with respect to frame vector fields:

$$\bar{\phi}_{,a} = \frac{f'}{\mathfrak{C} - 2X\mathfrak{D}}\phi_{,a}\,.$$

- Scalar field kinetic energy scalar:

$$\bar{X} = \left(\frac{f'}{\mathfrak{C} - 2X\mathfrak{D}}\right)^2 X\,.$$

- First scalar field derivative one-form:

$$\bar{\psi}_a = \frac{f'^2}{\mathfrak{C} - 2X\mathfrak{D}}\psi_a\,.$$

- Second scalar field derivative one-form:

$$\bar{\pi}_a = \frac{f'}{\mathfrak{C} - 2X\mathfrak{D}}\pi_a + \left(\frac{f'}{\mathfrak{C} - 2X\mathfrak{D}}\right)_{,\phi}\psi_a + \left(\frac{f'}{\mathfrak{C} - 2X\mathfrak{D}}\right)_{,X}\phi_{,a}\mathrm{d}X\,.$$

Outline

1. Introduction
2. Teleparallel disformal transformations
3. Construction of an invariant action
4. The quadratic class of actions
5. Conclusion

General structure of the action

- Structure of the action: gravity part S_g and matter part S_m such that:

$$S[\theta, \omega, \phi, \chi] = S_g[\theta, \omega, \phi] + S_m[\theta, \phi, \chi] .$$

[1] Assume that test particles follow the geodesics of the Levi-Civita connection. However, other assumptions are possible.

General structure of the action

- Structure of the action: gravity part S_g and matter part S_m such that:

$$S[\theta, \omega, \phi, \chi] = S_g[\theta, \omega, \phi] + S_m[\theta, \phi, \chi] .$$

- Conditions imposed on the matter action:
  - Invariance under local Lorentz transformations.
  - Form invariance under disformal transformations and scalar field redefinitions.
  - No direct coupling to the spin connection [1].

[1] Assume that test particles follow the geodesics of the Levi-Civita connection. However, other assumptions are possible.

General structure of the action

- Structure of the action: gravity part S_g and matter part S_m such that:

$$S[\theta, \omega, \phi, \chi] = S_g[\theta, \omega, \phi] + S_m[\theta, \phi, \chi] \,.$$

- Conditions imposed on the matter action:
  - Invariance under local Lorentz transformations.
  - Form invariance under disformal transformations and scalar field redefinitions.
  - No direct coupling to the spin connection [1].
- Conditions imposed on the gravity action:
  - Invariance under local Lorentz transformations.
  - Form invariance under disformal transformations and scalar field redefinitions.
  - Contains scalar quantities constructed from $T^a{}_{bc} = e_c \lrcorner e_b \lrcorner T^a$.
  - Is a linear combination of terms Q_k, with functions $F_k(\phi, X)$ as coefficients.

[1] Assume that test particles follow the geodesics of the Levi-Civita connection. However, other assumptions are possible.

Structure of the matter action

- Local Lorentz invariance ⇒ coupling only to the metric g:

$$S_m = S_m[g, \phi, \chi]\,.$$

⇒ Symmetric energy-momentum tensor.

Structure of the matter action

- Local Lorentz invariance ⇒ coupling only to the metric g:

$$S_m = S_m[g, \phi, \chi] \, .$$

⇒ Symmetric energy-momentum tensor.

- Form invariance under disformal transformations and scalar field redefinitions:

$$S_m = S_m[e^{2\alpha(\phi,X)} g + \beta(\phi, X) \mathrm{d}\phi \otimes \mathrm{d}\phi, \chi] \, .$$

⇒ Scalar field equation sourced by energy-momentum tensor.

Structure of the matter action

- Local Lorentz invariance $\Rightarrow$ coupling only to the metric g:

$$S_m = S_m[g, \phi, \chi]\,.$$

$\Rightarrow$ Symmetric energy-momentum tensor.

- Form invariance under disformal transformations and scalar field redefinitions:

$$S_m = S_m[e^{2\alpha(\phi,X)}g + \beta(\phi, X)\mathrm{d}\phi \otimes \mathrm{d}\phi, \chi]\,.$$

$\Rightarrow$ Scalar field equation sourced by energy-momentum tensor.

- Jordan frame - no direct coupling between matter and scalar field: $\alpha \equiv \beta \equiv 0$.

Torsion component $T^a{}_{bc}$ and its transformation

- Consider the torsion and its components in the tetrad basis:

$$A^{1a} = T^a\,, \quad A^{1a}{}_{bc} = T^a{}_{bc} = e_c \lrcorner e_b \lrcorner T^a\,.$$

Torsion component $T^a{}_{bc}$ and its transformation

- Consider the torsion and its components in the tetrad basis:

$$A^{1a} = T^a\,, \quad A^{1a}{}_{bc} = T^a{}_{bc} = e_c \lrcorner\, e_b \lrcorner\, T^a\,.$$

- Apply disformal transformation:

$$\bar{A}^{Ia}{}_{bc} = \bar{e}_c \lrcorner\, \bar{e}_b \lrcorner\, \bar{A}^{Ia} = \sum_{J=1}^{7} M^I{}_J(\phi, X) A^{Ja}{}_{bc}\,.$$

Torsion component $T^a{}_{bc}$ and its transformation

- Consider the torsion and its components in the tetrad basis:

$$A^{1a} = T^a\,, \quad A^{1a}{}_{bc} = T^a{}_{bc} = e_c \lrcorner e_b \lrcorner T^a\,.$$

- Apply disformal transformation:

$$\bar{A}^{Ia}{}_{bc} = \bar{e}_c \lrcorner \bar{e}_b \lrcorner \bar{A}^{Ia} = \sum_{J=1}^{7} M^I{}_J(\phi, X) A^{Ja}{}_{bc}\,.$$

- Newly appearing terms:

$$A^{2a}{}_{bc} = 2T^{ad}{}_{[c}\phi_{,b]}\phi_{,d}\,, \quad A^{3a}{}_{bc} = 2\phi_{,[b}\delta^a_{c]}\,, \quad A^{4a}{}_{bc} = 2X_{,[b}\delta^a_{c]}\,,$$

$$A^{5a}{}_{bc} = 2\eta^{ad}X_{,[b}\phi_{,c]}\phi_{,d}\,, \quad A^{6a}{}_{bc} = 2\phi_{,[c}e_{b]} \lrcorner \pi^a\,, \quad A^{7a}{}_{bc} = 2\eta^{de}\phi_{,d}X_{,e}\phi_{,[b}\delta^a_{c]}\,.$$

Torsion component $T^a{}_{bc}$ and its transformation

- Consider the torsion and its components in the tetrad basis:

$$A^{1a} = T^a\,, \quad A^{1a}{}_{bc} = T^a{}_{bc} = e_c \lrcorner e_b \lrcorner T^a\,.$$

- Apply disformal transformation:

$$\bar{A}^{Ia}{}_{bc} = \bar{e}_c \lrcorner \bar{e}_b \lrcorner \bar{A}^{Ia} = \sum_{J=1}^{7} M^I{}_J(\phi, X) A^{Ja}{}_{bc}\,.$$

- Newly appearing terms:

$$A^{2a}{}_{bc} = 2T^{ad}{}_{[c}\phi_{,b]}\phi_{,d}\,, \quad A^{3a}{}_{bc} = 2\phi_{,[b}\delta^a_{c]}\,, \quad A^{4a}{}_{bc} = 2X_{,[b}\delta^a_{c]}\,,$$
$$A^{5a}{}_{bc} = 2\eta^{ad}X_{,[b}\phi_{,c]}\phi_{,d}\,, \quad A^{6a}{}_{bc} = 2\phi_{,[c}e_{b]} \lrcorner \pi^a\,, \quad A^{7a}{}_{bc} = 2\eta^{de}\phi_{,d}X_{,e}\phi_{,[b}\delta^a_{c]}\,.$$

- Coefficients in the transformation (with $\mathfrak{E} = \mathfrak{C} - 2X\mathfrak{D}$):

$$M^1{}_1 = \frac{1}{\mathfrak{C}}\,, \; M^1{}_6 = -M^1{}_2 = \frac{\mathfrak{D}}{\mathfrak{C}\mathfrak{E}}\,, \; M^1{}_3 = \frac{\mathfrak{C}_{,\phi}}{\mathfrak{C}\mathfrak{E}}\,, \; M^1{}_4 = \frac{\mathfrak{C}_{,X}}{\mathfrak{C}^2}\,, \; M^1{}_5 = \frac{\mathfrak{C}\mathfrak{D}_{,X} - \mathfrak{D}\mathfrak{C}_{,X}}{\mathfrak{C}^2\mathfrak{E}}\,, \; M^1{}_7 = -\frac{\mathfrak{D}\mathfrak{C}_X}{\mathfrak{C}^2\mathfrak{E}}\,.$$

Transformation of resulting terms

- Define the functions (abbreviations):

$$\mathfrak{F} = \mathfrak{C} + 2X\mathfrak{D} - 2X\mathfrak{C}_{,X} + 4X^2\mathfrak{D}_{,X}\,, \quad \mathfrak{G} = \left(\frac{f'}{\mathfrak{C}}\right)_{,\phi} = \frac{f''}{\mathfrak{C} - 2X\mathfrak{D}} - \frac{f'(\mathfrak{C}_{,\phi} - 2X\mathfrak{D}_{,\phi})}{(\mathfrak{C} - 2X\mathfrak{D})^2}\,.$$

Transformation of resulting terms

- Define the functions (abbreviations):

$$\mathfrak{F} = \mathfrak{C} + 2X\mathfrak{D} - 2X\mathfrak{C}_{,X} + 4X^2\mathfrak{D}_{,X}\,, \quad \mathfrak{G} = \left(\frac{f'}{\mathfrak{E}}\right)_{,\phi} = \frac{f''}{\mathfrak{C} - 2X\mathfrak{D}} - \frac{f'(\mathfrak{C}_{,\phi} - 2X\mathfrak{D}_{,\phi})}{(\mathfrak{C} - 2X\mathfrak{D})^2}\,.$$

- Coefficients in the transformation of $A^{2a}{}_{bc}, \ldots, A^{7a}{}_{bc}$:

$$M^2{}_2 = \frac{f'^2}{\mathfrak{E}^3}\,, \quad M^2{}_3 = -\frac{2Xf'^2\mathfrak{C}_{,\phi}}{\mathfrak{C}\mathfrak{E}^3}\,, \quad M^6{}_5 = \frac{f'^2(2\mathfrak{D} - \mathfrak{C}_{,X} + 2X\mathfrak{D}_{,X})}{\mathfrak{C}\mathfrak{E}^3}\,, \quad M^2{}_6 = -\frac{2Xf'^2\mathfrak{D}}{\mathfrak{C}\mathfrak{E}^3}\,,$$

$$M^2{}_7 = \frac{f'^2\mathfrak{C}_{,X}}{\mathfrak{C}\mathfrak{E}^3}\,, \quad M^3{}_3 = \frac{f'}{\mathfrak{E}}\,, \quad M^5{}_5 = \frac{f'^4\mathfrak{F}}{\mathfrak{C}\mathfrak{E}^5}\,, \quad M^2{}_5 = \frac{f'^2(\mathfrak{C}_{,X} - 2X\mathfrak{D}_{,X})}{\mathfrak{C}\mathfrak{E}^3}\,, \quad M^6{}_6 = \frac{f'^2}{\mathfrak{C}\mathfrak{E}^2}\,,$$

$$M^4{}_3 = \frac{2Xf'\mathfrak{G}}{\mathfrak{E}^2}\,, \quad M^4{}_4 = \frac{f'^2\mathfrak{F}}{\mathfrak{C}\mathfrak{E}^3}\,, \quad M^4{}_7 = -\frac{f'^2\mathfrak{D}\mathfrak{F}}{\mathfrak{C}\mathfrak{E}^4}\,, \quad M^7{}_3 = -\frac{4X^2f'^3\mathfrak{G}}{\mathfrak{E}^4}\,, \quad M^7{}_7 = \frac{f'^4\mathfrak{F}}{\mathfrak{E}^6}\,.$$

Transformation of resulting terms

- Define the functions (abbreviations):

$$\mathfrak{F} = \mathfrak{C} + 2X\mathfrak{D} - 2X\mathfrak{C}_{,X} + 4X^2\mathfrak{D}_{,X}\,, \quad \mathfrak{G} = \left(\frac{f'}{\mathfrak{E}}\right)_{,\phi} = \frac{f''}{\mathfrak{C} - 2X\mathfrak{D}} - \frac{f'(\mathfrak{C}_{,\phi} - 2X\mathfrak{D}_{,\phi})}{(\mathfrak{C} - 2X\mathfrak{D})^2}\,.$$

- Coefficients in the transformation of $A^{2a}{}_{bc}, \ldots, A^{7a}{}_{bc}$:

$$M^2{}_2 = \frac{f'^2}{\mathfrak{E}^3}\,, \quad M^2{}_3 = -\frac{2Xf'^2\mathfrak{C}_{,\phi}}{\mathfrak{C}\mathfrak{E}^3}\,, \quad M^6{}_5 = \frac{f'^2(2\mathfrak{D} - \mathfrak{C}_{,X} + 2X\mathfrak{D}_{,X})}{\mathfrak{C}\mathfrak{E}^3}\,, \quad M^2{}_6 = -\frac{2Xf'^2\mathfrak{D}}{\mathfrak{C}\mathfrak{E}^3}\,,$$

$$M^2{}_7 = \frac{f'^2\mathfrak{C}_{,X}}{\mathfrak{C}\mathfrak{E}^3}\,, \quad M^3{}_3 = \frac{f'}{\mathfrak{E}}\,, \quad M^5{}_5 = \frac{f'^4\mathfrak{F}}{\mathfrak{C}\mathfrak{E}^5}\,, \quad M^2{}_5 = \frac{f'^2(\mathfrak{C}_{,X} - 2X\mathfrak{D}_{,X})}{\mathfrak{C}\mathfrak{E}^3}\,, \quad M^6{}_6 = \frac{f'^2}{\mathfrak{C}\mathfrak{E}^2}\,,$$

$$M^4{}_3 = \frac{2Xf'\mathfrak{G}}{\mathfrak{E}^2}\,, \quad M^4{}_4 = \frac{f'^2\mathfrak{F}}{\mathfrak{C}\mathfrak{E}^3}\,, \quad M^4{}_7 = -\frac{f'^2\mathfrak{D}\mathfrak{F}}{\mathfrak{C}\mathfrak{E}^4}\,, \quad M^7{}_3 = -\frac{4X^2f'^3\mathfrak{G}}{\mathfrak{E}^4}\,, \quad M^7{}_7 = \frac{f'^4\mathfrak{F}}{\mathfrak{E}^6}\,.$$

$\Rightarrow$ Transformation reproduces terms $A^{1a}{}_{bc}, \ldots, A^{7a}{}_{bc}$ with coefficients $M^I{}_J(\phi, X)$.

Disformally invariant gravitational action

- Consider action which is polynomial in building blocks:

$$S_g = \int_M \mathrm{vol}_\theta \sum_{N=0}^{\infty} \sum_{I_1=1}^{7} \cdots \sum_{I_N=1}^{7} H_{I_1 \cdots I_N a_1 \cdots a_N}{}^{b_1 \cdots b_N c_1 \cdots c_N}(\phi, X) A^{I_1 a_1}{}_{b_1 c_1} \cdots A^{I_N a_N}{}_{b_N c_N} .$$

Disformally invariant gravitational action

- Consider action which is polynomial in building blocks:

$$S_g = \int_M \mathrm{vol}_\theta \sum_{N=0}^{\infty} \sum_{I_1=1}^{7} \cdots \sum_{I_N=1}^{7} H_{I_1 \cdots I_N a_1 \cdots a_N}{}^{b_1 \cdots b_N c_1 \cdots c_N}(\phi, X) A^{I_1 a_1}{}_{b_1 c_1} \cdots A^{I_N a_N}{}_{b_N c_N} \,.$$

- Behavior under disformal transformation:

$$S_g = \int_M \mathrm{vol}_{\bar{\theta}} \sum_{N=0}^{\infty} \sum_{I_1=1}^{7} \cdots \sum_{I_N=1}^{7} \bar{H}_{I_1 \cdots I_N a_1 \cdots a_N}{}^{b_1 \cdots b_N c_1 \cdots c_N}(\bar{\phi}, \bar{X}) \bar{A}^{I_1 a_1}{}_{b_1 c_1} \cdots \bar{A}^{I_N a_N}{}_{b_N c_N} \,.$$

Disformally invariant gravitational action

- Consider action which is polynomial in building blocks:

$$S_g = \int_M \mathrm{vol}_\theta \sum_{N=0}^{\infty} \sum_{I_1=1}^{7} \cdots \sum_{I_N=1}^{7} H_{I_1 \cdots I_N a_1 \cdots a_N}{}^{b_1 \cdots b_N c_1 \cdots c_N}(\phi, X) A^{I_1 a_1}{}_{b_1 c_1} \cdots A^{I_N a_N}{}_{b_N c_N} .$$

- Behavior under disformal transformation:

$$S_g = \int_M \mathrm{vol}_{\bar\theta} \sum_{N=0}^{\infty} \sum_{I_1=1}^{7} \cdots \sum_{I_N=1}^{7} \bar{H}_{I_1 \cdots I_N a_1 \cdots a_N}{}^{b_1 \cdots b_N c_1 \cdots c_N}(\bar\phi, \bar{X}) \bar{A}^{I_1 a_1}{}_{b_1 c_1} \cdots \bar{A}^{I_N a_N}{}_{b_N c_N} .$$

- Relation between parameter functions:

$$H_{I_1 \cdots I_N a_1 \cdots a_N}{}^{b_1 \cdots b_N c_1 \cdots c_N} = \mathfrak{C}^3(\mathfrak{C} - 2X\mathfrak{D}) \sum_{J_1=1}^{7} \cdots \sum_{J_N=1}^{7} \bar{H}_{J_1 \cdots J_N a_1 \cdots a_N}{}^{b_1 \cdots b_N c_1 \cdots c_N} M^{J_1}{}_{I_1} \cdots M^{J_N}{}_{I_N} .$$

Outline

1. Introduction
2. Teleparallel disformal transformations
3. Construction of an invariant action
4. The quadratic class of actions
5. Conclusion

Lowest order terms in the general action

- Even number of indices needed to obtain a scalar $\Rightarrow$ N must be even.

Lowest order terms in the general action

- Even number of indices needed to obtain a scalar $\Rightarrow$ N must be even.
- Zeroth order term $N = 0$: function $H(\phi, X)$.

Lowest order terms in the general action

- Even number of indices needed to obtain a scalar $\Rightarrow$ N must be even.
- Zeroth order term $N = 0$: function $H(\phi, X)$.
- Second order terms defined by free functions $U_{IJ}(\phi, X)$, $V_{IJ}(\phi, X)$, $W_{IJ}(\phi, X)$:

$$H_{I_1 I_2 a_1 a_2}{}^{b_1 b_2 c_1 c_2} = U_{I_1 I_2}\eta_{a_1 a_2}\eta^{b_1 b_2}\eta^{c_1 c_2} + V_{I_1 I_2}\delta_{a_2}^{[c_1}\eta^{b_1][b_2}\delta_{a_1}^{c_2]} + W_{I_1 I_2}\delta_{a_1}^{[c_1}\eta^{b_1][b_2}\delta_{a_2}^{c_2]} \,.$$

Lowest order terms in the general action

- Even number of indices needed to obtain a scalar $\Rightarrow$ N must be even.
- Zeroth order term $N = 0$: function $H(\phi, X)$.
- Second order terms defined by free functions $U_{IJ}(\phi, X)$, $V_{IJ}(\phi, X)$, $W_{IJ}(\phi, X)$:

$$H_{I_1 I_2 a_1 a_2}{}^{b_1 b_2 c_1 c_2} = U_{I_1 I_2}\eta_{a_1 a_2}\eta^{b_1 b_2}\eta^{c_1 c_2} + V_{I_1 I_2}\delta_{a_2}^{[c_1}\eta^{b_1][b_2}\delta_{a_1}^{c_2]} + W_{I_1 I_2}\delta_{a_1}^{[c_1}\eta^{b_1][b_2}\delta_{a_2}^{c_2]} .$$

- Quadratic disformally invariant class of actions:

$$S_g = \int_M \mathrm{vol}_\theta \left[H(\phi, X) + \sum_{I_1=1}^{7}\sum_{I_2=1}^{7} H_{I_1 I_2 a_1 a_2}{}^{b_1 b_2 c_1 c_2}(\phi, X) A^{I_1 a_1}{}_{b_1 c_1} A^{I_2 a_2}{}_{b_2 c_2} \right] .$$

Lowest order terms in the general action

- Even number of indices needed to obtain a scalar $\Rightarrow$ N must be even.
- Zeroth order term $N = 0$: function $H(\phi, X)$.
- Second order terms defined by free functions $U_{IJ}(\phi, X)$, $V_{IJ}(\phi, X)$, $W_{IJ}(\phi, X)$:

$$H_{I_1 I_2 a_1 a_2}{}^{b_1 b_2 c_1 c_2} = U_{I_1 I_2} \eta_{a_1 a_2} \eta^{b_1 b_2} \eta^{c_1 c_2} + V_{I_1 I_2} \delta^{[c_1}_{a_2} \eta^{b_1][b_2} \delta^{c_2]}_{a_1} + W_{I_1 I_2} \delta^{[c_1}_{a_1} \eta^{b_1][b_2} \delta^{c_2]}_{a_2} .$$

- Quadratic disformally invariant class of actions:

$$S_g = \int_M \mathrm{vol}_\theta \left[H(\phi, X) + \sum_{I_1=1}^{7} \sum_{I_2=1}^{7} H_{I_1 I_2 a_1 a_2}{}^{b_1 b_2 c_1 c_2}(\phi, X) A^{I_1 a_1}{}_{b_1 c_1} A^{I_2 a_2}{}_{b_2 c_2} \right] .$$

- Explicit form of all linearly independent terms in this action?

Terms in the quadratic class of actions

- Terms quadratic in the torsion only:

$$Q_1 = T^{abc}T_{abc}\,, \quad Q_2 = T^{abc}T_{cba}\,, \quad Q_3 = T^{a}{}_{ac}T_{b}{}^{bc}\,.$$

Terms in the quadratic class of actions

- Terms quadratic in the torsion only:

$$Q_1 = T^{abc}T_{abc}\,, \quad Q_2 = T^{abc}T_{cba}\,, \quad Q_3 = T^a{}_{ac}T_b{}^{bc}\,.$$

- Terms quadratic in the torsion which contain the scalar field:

$$Q_4 = T^{abe}T^{cd}{}_e\phi_{,a}\phi_{,b}\phi_{,c}\phi_{,d}\,, \quad Q_5 = T^{abc}T^d{}_{dc}\phi_{,a}\phi_{,b}\,, \quad Q_6 = T^{cda}T_{cd}{}^b\phi_{,a}\phi_{,b}\,,$$

$$Q_7 = T^{cda}T_{dc}{}^b\phi_{,a}\phi_{,b}\,, \quad Q_8 = T^{cda}T^b{}_{cd}\phi_{,a}\phi_{,b}\,, \quad Q_9 = T^{acd}T^b{}_{cd}\phi_{,a}\phi_{,b}\,.$$

Terms in the quadratic class of actions

- Terms quadratic in the torsion only:

$$Q_1 = T^{abc}T_{abc}\,, \quad Q_2 = T^{abc}T_{cba}\,, \quad Q_3 = T^a{}_{ac}T_b{}^{bc}\,.$$

- Terms quadratic in the torsion which contain the scalar field:

$$Q_4 = T^{abe}T^{cd}{}_e\phi_{,a}\phi_{,b}\phi_{,c}\phi_{,d}\,, \quad Q_5 = T^{abc}T^d{}_{dc}\phi_{,a}\phi_{,b}\,, \quad Q_6 = T^{cda}T_{cd}{}^b\phi_{,a}\phi_{,b}\,,$$
$$Q_7 = T^{cda}T_{dc}{}^b\phi_{,a}\phi_{,b}\,, \quad Q_8 = T^{cda}T^b{}_{cd}\phi_{,a}\phi_{,b}\,, \quad Q_9 = T^{acd}T^b{}_{cd}\phi_{,a}\phi_{,b}\,.$$

- Terms linear in the torsion:

$$Q_{10} = T_a{}^{ab}\phi_{,b}\,, \quad Q_{11} = T^{abc}e_a \lrcorner\, \pi_b\phi_{,c}\,, \quad Q_{12} = T_a{}^{ab}X_{,b}\,, \quad Q_{13} = T^{abc}\phi_{,a}\phi_{,b}X_{,c}\,.$$

Terms in the quadratic class of actions

- Terms quadratic in the torsion only:

$$Q_1 = T^{abc}T_{abc}\,, \quad Q_2 = T^{abc}T_{cba}\,, \quad Q_3 = T^a{}_{ac}T_b{}^{bc}\,.$$

- Terms quadratic in the torsion which contain the scalar field:

$$Q_4 = T^{abe}T^{cd}{}_e\phi_{,a}\phi_{,b}\phi_{,c}\phi_{,d}\,, \quad Q_5 = T^{abc}T^d{}_{dc}\phi_{,a}\phi_{,b}\,, \quad Q_6 = T^{cda}T_{cd}{}^b\phi_{,a}\phi_{,b}\,,$$

$$Q_7 = T^{cda}T_{dc}{}^b\phi_{,a}\phi_{,b}\,, \quad Q_8 = T^{cda}T^b{}_{cd}\phi_{,a}\phi_{,b}\,, \quad Q_9 = T^{acd}T^b{}_{cd}\phi_{,a}\phi_{,b}\,.$$

- Terms linear in the torsion:

$$Q_{10} = T_a{}^{ab}\phi_{,b}\,, \quad Q_{11} = T^{abc}e_a \lrcorner \pi_b\phi_{,c}\,, \quad Q_{12} = T_a{}^{ab}X_{,b}\,, \quad Q_{13} = T^{abc}\phi_{,a}\phi_{,b}X_{,c}\,.$$

- Terms involving the scalar field only:

$$Q_{14} = X_{,a}X^{,a}\,, \quad Q_{15} = X_{,a}\phi^{,a}\,, \quad Q_{16} = (e_a \lrcorner \pi^b)(e_b \lrcorner \pi^a)\,, \quad Q_{17} = e_a \lrcorner \pi^a\,.$$

Terms in the quadratic class of actions

- Terms quadratic in the torsion only:

$$Q_1 = T^{abc}T_{abc}\,, \quad Q_2 = T^{abc}T_{cba}\,, \quad Q_3 = T^a{}_{ac}T_b{}^{bc}\,.$$

- Terms quadratic in the torsion which contain the scalar field:

$$Q_4 = T^{abe}T^{cd}{}_e\phi_{,a}\phi_{,b}\phi_{,c}\phi_{,d}\,, \quad Q_5 = T^{abc}T^d{}_{dc}\phi_{,a}\phi_{,b}\,, \quad Q_6 = T^{cda}T_{cd}{}^b\phi_{,a}\phi_{,b}\,,$$
$$Q_7 = T^{cda}T_{dc}{}^b\phi_{,a}\phi_{,b}\,, \quad Q_8 = T^{cda}T^b{}_{cd}\phi_{,a}\phi_{,b}\,, \quad Q_9 = T^{acd}T^b{}_{cd}\phi_{,a}\phi_{,b}\,.$$

- Terms linear in the torsion:

$$Q_{10} = T_a{}^{ab}\phi_{,b}\,, \quad Q_{11} = T^{abc}e_a \lrcorner \pi_b\phi_{,c}\,, \quad Q_{12} = T_a{}^{ab}X_{,b}\,, \quad Q_{13} = T^{abc}\phi_{,a}\phi_{,b}X_{,c}\,.$$

- Terms involving the scalar field only:

$$Q_{14} = X_{,a}X^{,a}\,, \quad Q_{15} = X_{,a}\phi^{,a}\,, \quad Q_{16} = (e_a \lrcorner \pi^b)(e_b \lrcorner \pi^a)\,, \quad Q_{17} = e_a \lrcorner \pi^a\,.$$

- Products of the simple terms:

$$Q_{18} = Q_{15}Q_{17}\,,\ Q_{19} = Q_{10}Q_{17}\,,\ Q_{20} = Q_{10}Q_{15}\,,\ Q_{21} = Q_{17}^2\,,\ Q_{22} = Q_{15}^2\,,\ Q_{23} = Q_{10}^2\,.$$

Full form of the at most quadratic action

- Term Q_{23} appears only in certain linear combinations ⇒ redefinition of terms:

$$\tilde{Q}_4 = Q_4 - 2XQ_{23}\,, \quad \tilde{Q}_5 = Q_5 + Q_{23}\,, \quad \tilde{Q}_7 = Q_7 - Q_{23}\,,$$
$$\tilde{Q}_9 = Q_9 + 2Q_{23}\,, \quad \tilde{Q}_{11} = Q_{11} + Q_{23}\,, \quad \tilde{Q}_{16} = Q_{16} + Q_{23}\,.$$

Full form of the at most quadratic action

- Term Q_{23} appears only in certain linear combinations ⇒ redefinition of terms:

$$\tilde{Q}_4 = Q_4 - 2XQ_{23}\,, \quad \tilde{Q}_5 = Q_5 + Q_{23}\,, \quad \tilde{Q}_7 = Q_7 - Q_{23}\,,$$
$$\tilde{Q}_9 = Q_9 + 2Q_{23}\,, \quad \tilde{Q}_{11} = Q_{11} + Q_{23}\,, \quad \tilde{Q}_{16} = Q_{16} + Q_{23}\,.$$

- Introduce zeroth order term $\tilde{Q}_0 = 1$.

Full form of the at most quadratic action

- Term Q_{23} appears only in certain linear combinations $\Rightarrow$ redefinition of terms:

$$\tilde{Q}_4 = Q_4 - 2XQ_{23}\,, \quad \tilde{Q}_5 = Q_5 + Q_{23}\,, \quad \tilde{Q}_7 = Q_7 - Q_{23}\,,$$
$$\tilde{Q}_9 = Q_9 + 2Q_{23}\,, \quad \tilde{Q}_{11} = Q_{11} + Q_{23}\,, \quad \tilde{Q}_{16} = Q_{16} + Q_{23}\,.$$

- Introduce zeroth order term $\tilde{Q}_0 = 1$.

$\Rightarrow$ Transformation behavior of constructed terms:

$$\bar{\tilde{Q}}_k = \sum_{l=0}^{22} \tilde{M}_{kl}(\phi, X)\tilde{Q}_l\,.$$

Full form of the at most quadratic action

- Term Q_{23} appears only in certain linear combinations $\Rightarrow$ redefinition of terms:

$$\tilde{Q}_4 = Q_4 - 2XQ_{23}\,, \quad \tilde{Q}_5 = Q_5 + Q_{23}\,, \quad \tilde{Q}_7 = Q_7 - Q_{23}\,,$$
$$\tilde{Q}_9 = Q_9 + 2Q_{23}\,, \quad \tilde{Q}_{11} = Q_{11} + Q_{23}\,, \quad \tilde{Q}_{16} = Q_{16} + Q_{23}\,.$$

- Introduce zeroth order term $\tilde{Q}_0 = 1$.

$\Rightarrow$ Transformation behavior of constructed terms:

$$\bar{\tilde{Q}}_k = \sum_{l=0}^{22} \tilde{M}_{kl}(\phi, X)\tilde{Q}_l\,.$$

$\Rightarrow$ General form of the disformally invariant quadratic class of actions:

$$S_g = \int_M \mathrm{vol}_\theta \sum_{k=0}^{22} F_k(\phi, X)\tilde{Q}_k\,.$$

Outline

1 Introduction

2 Teleparallel disformal transformations

3 Construction of an invariant action

4 The quadratic class of actions

5 Conclusion

Summary

- Properties of the constructed actions:
  - Invariance under local Lorentz transformations.
  - Form invariance under disformal transformations and scalar field redefinitions.
  - No direct coupling between matter spin connection.

Summary

- Properties of the constructed actions:
  - Invariance under local Lorentz transformations.
  - Form invariance under disformal transformations and scalar field redefinitions.
  - No direct coupling between matter spin connection.
- Polynomial class of actions:
  - Constructed from 7 tensors involving torsion and scalar field.
  - Terms are polynomial in these tensors with coefficients $\sim f(\phi, X)$.
  - May consider arbitrary orders.

Summary

- Properties of the constructed actions:
  - Invariance under local Lorentz transformations.
  - Form invariance under disformal transformations and scalar field redefinitions.
  - No direct coupling between matter spin connection.
- Polynomial class of actions:
  - Constructed from 7 tensors involving torsion and scalar field.
  - Terms are polynomial in these tensors with coefficients $\sim f(\phi, X)$.
  - May consider arbitrary orders.
- Quadratic class of actions:
  - Consider only polynomial terms of up to quadratic order.
  - General action is linear combination of 23 different terms.
  - Does not contain equivalent of Horndeski gravity ($\mathcal{L}_5$ needs quartic order).

Outlook

- Theories avoiding the appearance of Ostrogradsky ghosts?
  - Find subclass of theories with second order field equations.
  - Study invariance under special disformal transformations.
  - Use general disformal transformations to generate ghost-free higher order theories.
  - Perform Hamiltonian analysis and study ghosts and degrees of freedom.

Outlook

- Theories avoiding the appearance of Ostrogradsky ghosts?
  - Find subclass of theories with second order field equations.
  - Study invariance under special disformal transformations.
  - Use general disformal transformations to generate ghost-free higher order theories.
  - Perform Hamiltonian analysis and study ghosts and degrees of freedom.
- Phenomenological aspects?
  - Cosmology - dynamics of FLRW universe and perturbations.
  - Spherically symmetric solutions - black holes, their shadows and orbits.
  - Screening mechanisms (Vainshtein etc.) and strong coupling issues.
  - Post-Newtonian limit and solar system consistency.
  - Gravitational wave - speed and polarizations.

Outlook

- Theories avoiding the appearance of Ostrogradsky ghosts?
  - Find subclass of theories with second order field equations.
  - Study invariance under special disformal transformations.
  - Use general disformal transformations to generate ghost-free higher order theories.
  - Perform Hamiltonian analysis and study ghosts and degrees of freedom.
- Phenomenological aspects?
  - Cosmology - dynamics of FLRW universe and perturbations.
  - Spherically symmetric solutions - black holes, their shadows and orbits.
  - Screening mechanisms (Vainshtein etc.) and strong coupling issues.
  - Post-Newtonian limit and solar system consistency.
  - Gravitational wave - speed and polarizations.
- Extensions:
  - Multiple scalar fields.
  - Extended disformal transformations and degenerate higher order theories.
  - Arbitrary functions of scalars obtained from contracting terms $A^{I\,a}{}_{bc}$.

Outlook

- Theories avoiding the appearance of Ostrogradsky ghosts?
  - Find subclass of theories with second order field equations.
  - Study invariance under special disformal transformations.
  - Use general disformal transformations to generate ghost-free higher order theories.
  - Perform Hamiltonian analysis and study ghosts and degrees of freedom.
- Phenomenological aspects?
  - Cosmology - dynamics of FLRW universe and perturbations.
  - Spherically symmetric solutions - black holes, their shadows and orbits.
  - Screening mechanisms (Vainshtein etc.) and strong coupling issues.
  - Post-Newtonian limit and solar system consistency.
  - Gravitational wave - speed and polarizations.
- Extensions:
  - Multiple scalar fields.
  - Extended disformal transformations and degenerate higher order theories.
  - Arbitrary functions of scalars obtained from contracting terms $A^{I\,a}{}_{bc}$.

MH, "Disformal Transformations in Scalar-Torsion Gravity," arXiv:1905.00451 [gr-qc]