The universe as a whole in teleparallel gravity

Manuel Hohmann

Laboratory of Theoretical Physics - Institute of Physics - University of Tartu
Center of Excellence “The Dark Side of the Universe”

European Union
European Regional
Development Fund

Investing
in your future

Tuorla-Tartu annual meeting 2018

Outline

Outline

Motivation

- Open questions in cosmology and gravity:
  - Accelerating phases in the history of the Universe - dark energy, inflation?
  - Relation between gravity, quantum theory and gauge theories?

Motivation

- Open questions in cosmology and gravity:
  - Accelerating phases in the history of the Universe - dark energy, inflation?
  - Relation between gravity, quantum theory and gauge theories?
- Teleparallel gravity and symmetric teleparallel gravity:
  - Based on a different (flat) connection - gravity is *not* mediated by curvature.
  - Teleparallel gravity: interaction is mediated by the *torsion*.
  - Symmetric teleparallel gravity: interaction is mediated by the *non-metricity*.

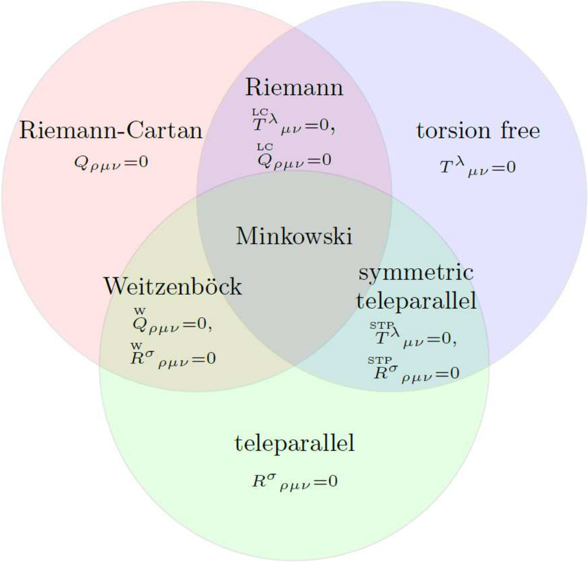
Motivation

- Open questions in cosmology and gravity:
  - Accelerating phases in the history of the Universe - dark energy, inflation?
  - Relation between gravity, quantum theory and gauge theories?
- Teleparallel gravity and symmetric teleparallel gravity:
  - Based on a different (flat) connection - gravity is *not* mediated by curvature.
  - Teleparallel gravity: interaction is mediated by the *torsion*.
  - Symmetric teleparallel gravity: interaction is mediated by the *non-metricity*.
- Modified gravity theories based on (symmetric) teleparallel gravity:
  - Contains $f(T)$ gravity [Bengochea, Ferraro '09] and $f(Q)$ gravity [Beltran Jimenez, Heisenberg, Koivisto '17].
  - Contains new GR [Hayashi, Shirafuji '79] and newer GR [Beltran Jimenez, Heisenberg, Koivisto '17].
  - Contains teleparallel dark energy [Geng '11].
  - Contains scalar-torsion gravity in covariant formulation [MH, Järv, Ualikhanova '18].
  - Contains scalar-non-metricity gravity [Järv, Rünkla, Saal, Vilson '18].

Motivation

- Open questions in cosmology and gravity:
  - Accelerating phases in the history of the Universe - dark energy, inflation?
  - Relation between gravity, quantum theory and gauge theories?
- Teleparallel gravity and symmetric teleparallel gravity:
  - Based on a different (flat) connection - gravity is *not* mediated by curvature.
  - Teleparallel gravity: interaction is mediated by the *torsion*.
  - Symmetric teleparallel gravity: interaction is mediated by the *non-metricity*.
- Modified gravity theories based on (symmetric) teleparallel gravity:
  - Contains $f(T)$ gravity [Bengochea, Ferraro '09] and $f(Q)$ gravity [Beltran Jimenez, Heisenberg, Koivisto '17].
  - Contains new GR [Hayashi, Shirafuji '79] and newer GR [Beltran Jimenez, Heisenberg, Koivisto '17].
  - Contains teleparallel dark energy [Geng '11].
  - Contains scalar-torsion gravity in covariant formulation [MH, Järv, Ualikhanova '18].
  - Contains scalar-non-metricity gravity [Järv, Rünkla, Saal, Vilson '18].
- Teleparallel cosmology - how to describe the Universe as a whole:
  - Flat cosmology allows for de Sitter attractors [MH, Järv, Ualikhanova '17].
  - Make use of cosmological symmetry in order to find further solutions?
  - Modified Friedmann equations for non-flat models?
  - How to distinguish and exclude models based on cosmological observables?

The trinity of geometric models of gravity



Outline

Gravity in the teleparallel language vs curvature gravity

Curvature gravity	**Torsion gravity**
Fundamental fields	
Metric $g_{\mu\nu}$	Tetrad $\theta^a{}_\mu$ & inverse $e_a{}^\mu$ Spin connection $\dot{\omega}^a{}_{b\mu}$
Constraints	
-	$\partial_{[\mu}\dot{\omega}^a{}_{\lvert b \rvert\nu]} + \dot{\omega}^a{}_{c[\mu}\dot{\omega}^c{}_{\lvert b \rvert\nu]} = 0$
Derived quantities	
Connection $\mathring{\Gamma}^\rho{}_{\mu\nu} = \frac{1}{2}g^{\rho\sigma}\left(g_{\mu\sigma,\nu} + g_{\nu\sigma,\mu} - g_{\mu\nu,\sigma}\right)$	Metric $g_{\mu\nu} = \eta_{ab}\theta^a{}_\mu\theta^b{}_\nu$ Connection $\dot{\Gamma}^\rho{}_{\mu\nu} = e_a{}^\rho\left(\partial_\nu\theta^a{}_\mu + \theta^b{}_\mu\dot{\omega}^a{}_{b\nu}\right)$
Quantity mediating gravity	
Curvature $\mathring{R}^\mu{}_{\nu\rho\sigma} = 2\left(\partial_{[\rho}\mathring{\Gamma}^\mu{}_{\lvert\nu\rvert\sigma]} + \mathring{\Gamma}^\mu{}_{\tau[\rho}\mathring{\Gamma}^\tau{}_{\lvert\nu\rvert\sigma]}\right)$	Torsion $\dot{T}^\rho{}_{\mu\nu} = 2\dot{\Gamma}^\rho{}_{[\nu\mu]}$
Vanishing quantities	
$\mathring{T}^\rho{}_{\mu\nu} = 2\mathring{\Gamma}^\rho{}_{[\nu\mu]} = 0$	$\dot{R}^\mu{}_{\nu\rho\sigma} = 2\left(\partial_{[\rho}\dot{\Gamma}^\mu{}_{\lvert\nu\rvert\sigma]} + \dot{\Gamma}^\mu{}_{\tau[\rho}\dot{\Gamma}^\tau{}_{\lvert\nu\rvert\sigma]}\right) = 0$

Modified teleparallel gravity: $f(T)$ theory action and field equations

- Gravitational action [Bengochea, Ferraro '09]:

$$S = \frac{1}{2\kappa^2}\int_M f(T)\,\theta\,d^4x + S_m[\theta^a, \chi^I]\,.$$

Modified teleparallel gravity: $f(T)$ theory action and field equations

- Gravitational action [Bengochea, Ferraro '09]:

$$S = \frac{1}{2\kappa^2}\int_M f(T)\,\theta\,d^4x + S_m[\theta^a, \chi^I]\,.$$

- Field equations:
  - Symmetric part of the tetrad field equations:

$$\frac{1}{2}fg_{\mu\nu} + \overset{\circ}{\nabla}_\rho\left(f_T S_{(\mu\nu)}{}^\rho\right) - \frac{1}{2}f_T S_{(\mu}{}^{\rho\sigma}T_{\nu)\rho\sigma} = \kappa^2\Theta_{\mu\nu}\,,$$

  - Antisymmetric part of the tetrad field equations = connection equations:

$$\partial_{[\rho}f_T T^\rho{}_{\mu\nu]} = 0\,.$$

Modified teleparallel gravity: $f(T)$ theory action and field equations

- Gravitational action [Bengochea, Ferraro '09]:

$$S = \frac{1}{2\kappa^2}\int_M f(T)\,\theta\,d^4x + S_m[\theta^a, \chi^I]\,.$$

- Field equations:
  - Symmetric part of the tetrad field equations:

$$\frac{1}{2}fg_{\mu\nu} + \overset{\circ}{\nabla}_\rho\left(f_T S_{(\mu\nu)}{}^\rho\right) - \frac{1}{2}f_T S_{(\mu}{}^{\rho\sigma}T_{\nu)\rho\sigma} = \kappa^2\Theta_{\mu\nu}\,,$$

  - Antisymmetric part of the tetrad field equations = connection equations:

$$\partial_{[\rho}f_T T^\rho{}_{\mu\nu]} = 0\,.$$

- Terms appearing in the action and field equations:
  - Superpotential: $S_\rho{}^{\mu\nu} = \frac{1}{2}\left(T^{\nu\mu}{}_\rho + T_\rho{}^{\mu\nu} - T^{\mu\nu}{}_\rho\right) - \delta^\mu_\rho T_\sigma{}^{\sigma\nu} + \delta^\nu_\rho T_\sigma{}^{\sigma\mu}$.

Modified teleparallel gravity: $f(T)$ theory action and field equations

- Gravitational action [Bengochea, Ferraro '09]:

$$S = \frac{1}{2\kappa^2}\int_M f(T)\,\theta\,d^4x + S_m[\theta^a, \chi^I]\,.$$

- Field equations:
  - Symmetric part of the tetrad field equations:

$$\frac{1}{2}fg_{\mu\nu} + \overset{\circ}{\nabla}_\rho\left(f_T S_{(\mu\nu)}{}^\rho\right) - \frac{1}{2}f_T S_{(\mu}{}^{\rho\sigma}T_{\nu)\rho\sigma} = \kappa^2\Theta_{\mu\nu}\,,$$

  - Antisymmetric part of the tetrad field equations = connection equations:

$$\partial_{[\rho}f_T T^\rho{}_{\mu\nu]} = 0\,.$$

- Terms appearing in the action and field equations:
  - Superpotential: $S_\rho{}^{\mu\nu} = \frac{1}{2}\left(T^{\nu\mu}{}_\rho + T_\rho{}^{\mu\nu} - T^{\mu\nu}{}_\rho\right) - \delta^\mu_\rho T_\sigma{}^{\sigma\nu} + \delta^\nu_\rho T_\sigma{}^{\sigma\mu}$.
  - Torsion scalar: $T = \frac{1}{2}T^\rho{}_{\mu\nu}S_\rho{}^{\mu\nu}$.

Modified teleparallel gravity: $f(T)$ theory action and field equations

- Gravitational action [Bengochea, Ferraro '09]:

$$S = \frac{1}{2\kappa^2}\int_M f(T)\,\theta\, d^4x + S_m[\theta^a, \chi^I]\,.$$

- Field equations:
  - Symmetric part of the tetrad field equations:

$$\frac{1}{2} f g_{\mu\nu} + \overset{\circ}{\nabla}_\rho \left(f_T S_{(\mu\nu)}{}^\rho\right) - \frac{1}{2} f_T S_{(\mu}{}^{\rho\sigma} T_{\nu)\rho\sigma} = \kappa^2 \Theta_{\mu\nu}\,,$$

  - Antisymmetric part of the tetrad field equations = connection equations:

$$\partial_{[\rho} f_T T^\rho{}_{\mu\nu]} = 0\,.$$

- Terms appearing in the action and field equations:
  - Superpotential: $S_\rho{}^{\mu\nu} = \frac{1}{2}\left(T^{\nu\mu}{}_\rho + T_\rho{}^{\mu\nu} - T^{\mu\nu}{}_\rho\right) - \delta^\mu_\rho T_\sigma{}^{\sigma\nu} + \delta^\nu_\rho T_\sigma{}^{\sigma\mu}$.
  - Torsion scalar: $T = \frac{1}{2} T^\rho{}_{\mu\nu} S_\rho{}^{\mu\nu}$.
  - Energy-momentum tensor $\Theta_{\mu\nu}$ derived from the matter part S_m of the action.

Spatially flat ($k = 0$) $f(T)$ cosmology as a dynamical system

- Ansatz for spatially flat ($k = 0$) cosmology:

$$\theta^a{}_\mu = \mathrm{diag}\,(1, a(t), a(t), a(t))\,, \quad \overset{\bullet}{\omega}{}^a{}_{b\mu} = 0 \quad \Rightarrow \quad g_{\mu\nu}dx^\mu dx^\nu = dt^2 - a^2(t)\delta_{ij}dx^i dx^j\,.$$

Spatially flat ($k = 0$) $f(T)$ cosmology as a dynamical system

- Ansatz for spatially flat ($k = 0$) cosmology:

$$\theta^a{}_\mu = \mathrm{diag}\,(1, a(t), a(t), a(t))\,, \quad \overset{\bullet}{\omega}{}^a{}_{b\mu} = 0 \quad \Rightarrow \quad g_{\mu\nu}dx^\mu dx^\nu = dt^2 - a^2(t)\delta_{ij}dx^i dx^j\,.$$

- Ansatz for perfect fluid matter:

$$\Theta^{\mu\nu} = (\rho + p)u^\mu u^\nu + pg^{\mu\nu}\,, \quad u^\mu = (1, 0, 0, 0)\,.$$

Spatially flat ($k = 0$) $f(T)$ cosmology as a dynamical system

- Ansatz for spatially flat ($k = 0$) cosmology:

$$\theta^a{}_\mu = \mathrm{diag}\,(1, a(t), a(t), a(t))\,, \quad \overset{\bullet}{\omega}{}^a{}_{b\mu} = 0 \quad \Rightarrow \quad g_{\mu\nu}dx^\mu dx^\nu = dt^2 - a^2(t)\delta_{ij}dx^i dx^j\,.$$

- Ansatz for perfect fluid matter:

$$\Theta^{\mu\nu} = (\rho + p)u^\mu u^\nu + pg^{\mu\nu}\,, \quad u^\mu = (1, 0, 0, 0)\,.$$

- Cosmological field equations using $T = -6H^2$:

$$12H^2 f_T + f = 2\kappa^2\rho\,,$$

$$48H^2\dot{H}f_{TT} - (12H^2 + 4\dot{H})f_T - f = 2\kappa^2 p\,,$$

Spatially flat ($k = 0$) $f(T)$ cosmology as a dynamical system

- Ansatz for spatially flat ($k = 0$) cosmology:

$$\theta^a{}_\mu = \mathrm{diag}\,(1, a(t), a(t), a(t))\,, \quad \overset{\bullet}{\omega}{}^a{}_{b\mu} = 0 \quad \Rightarrow \quad g_{\mu\nu}dx^\mu dx^\nu = dt^2 - a^2(t)\delta_{ij}dx^i dx^j\,.$$

- Ansatz for perfect fluid matter:

$$\Theta^{\mu\nu} = (\rho + p)u^\mu u^\nu + pg^{\mu\nu}\,, \quad u^\mu = (1, 0, 0, 0)\,.$$

- Cosmological field equations using $T = -6H^2$:

$$12H^2 f_T + f = 2\kappa^2\rho\,,$$
$$48H^2\dot{H}f_{TT} - (12H^2 + 4\dot{H})f_T - f = 2\kappa^2 p\,,$$

- Consider mixture of dust and radiation matter:

$$\rho = \rho_m + \rho_r\,, \quad p = p_m + p_r\,, \quad p_m = 0\,, \quad p_r = \frac{1}{3}\rho_r\,.$$

Spatially flat ($k = 0$) $f(T)$ cosmology as a dynamical system

- Ansatz for spatially flat ($k = 0$) cosmology:

$$\theta^a{}_\mu = \mathrm{diag}\,(1, a(t), a(t), a(t))\,, \quad \overset{\bullet}{\omega}{}^a{}_{b\mu} = 0 \quad \Rightarrow \quad g_{\mu\nu}dx^\mu dx^\nu = dt^2 - a^2(t)\delta_{ij}dx^i dx^j\,.$$

- Ansatz for perfect fluid matter:

$$\Theta^{\mu\nu} = (\rho + p)u^\mu u^\nu + pg^{\mu\nu}\,, \quad u^\mu = (1, 0, 0, 0)\,.$$

- Cosmological field equations using $T = -6H^2$:

$$12H^2 f_T + f = 2\kappa^2\rho\,,$$
$$48H^2\dot{H}f_{TT} - (12H^2 + 4\dot{H})f_T - f = 2\kappa^2 p\,,$$

- Consider mixture of dust and radiation matter:

$$\rho = \rho_m + \rho_r\,, \quad p = p_m + p_r\,, \quad p_m = 0\,, \quad p_r = \frac{1}{3}\rho_r\,.$$

- Cosmological dynamics as a dynamical system [MH, Järv, Ualikhanova '17]:

$$W(H) = 12H^2 f_T + f\,, \quad X = \frac{\rho_r}{\rho_r + \rho_m} \quad \Rightarrow \quad \dot{X} = HX(X - 1)\,, \quad \dot{H} = -\frac{(X + 3)H}{(\ln W)_H}\,.$$

Example: $f(T) = T + \alpha(-T)^n$ cosmology and evolution

- Consider simple power law model:

$$f(T) = T + \alpha(-T)^n .$$

Example: $f(T) = T + \alpha(-T)^n$ cosmology and evolution

- Consider simple power law model:

$$f(T) = T + \alpha(-T)^n\,.$$

- Function in dynamical system:

$$W = 6H^2 + (1-2n)\alpha(6H^2)^n\,.$$

Example: $f(T) = T + \alpha(-T)^n$ cosmology and evolution

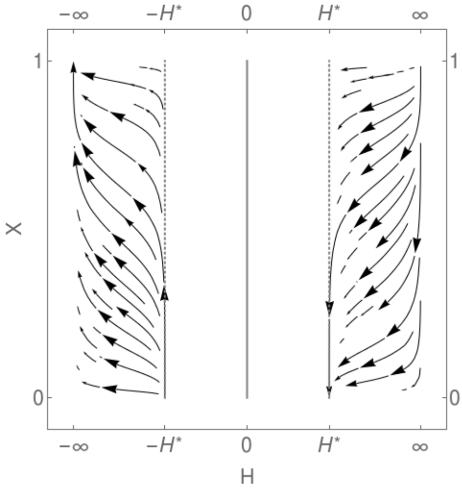
- Consider simple power law model:

$$f(T) = T + \alpha(-T)^n .$$

- Function in dynamical system:

$$W = 6H^2 + (1 - 2n)\alpha(6H^2)^n .$$

- For $\alpha > 0, \frac{1}{2} < n < 1$ or $\alpha < 0, n < \frac{1}{2}$:
  - Big bang at $H = \infty, X = 1$.
  - Transition from $\ddot{a} < 0$ to $\ddot{a} > 0$.
  - De Sitter attractor at $H = H^\star, X = 0$.
  - Phantom or non-phantom, no crossing.



The non-flat case: $k = 1$ cosmology

- Ansatz for $k = 1$ tetrad:

$$\theta^a{}_\mu = \operatorname{diag}\left(1, \frac{a(t)}{\sqrt{1 - r^2}}, a(t)r, a(t)r\sin\vartheta\right).$$

The non-flat case: $k = 1$ cosmology

- Ansatz for $k = 1$ tetrad:

$$\theta^a{}_\mu = \mathrm{diag}\left(1, \frac{a(t)}{\sqrt{1-r^2}}, a(t)r, a(t)r\sin\vartheta\right) .$$

- Resulting FLRW metric:

$$g_{\mu\nu}dx^\mu dx^\nu = dt^2 - a^2(t)\left[\frac{dr^2}{1-r^2} + r^2(d\vartheta^2 + \sin^2\vartheta d\varphi^2)\right] .$$

The non-flat case: $k = 1$ cosmology

- Ansatz for $k = 1$ tetrad:

$$\theta^a{}_\mu = \operatorname{diag}\left(1, \frac{a(t)}{\sqrt{1-r^2}}, a(t)r, a(t)r\sin\vartheta\right).$$

- Resulting FLRW metric:

$$g_{\mu\nu}dx^\mu dx^\nu = dt^2 - a^2(t)\left[\frac{dr^2}{1-r^2} + r^2(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2)\right].$$

- Solve antisymmetric part of the field equations using non-vanishing spin connection:

$$\overset{\bullet}{\omega}{}^1{}_{2\vartheta} = -\overset{\bullet}{\omega}{}^2{}_{1\vartheta} = -\sqrt{1-r^2}\,, \quad \overset{\bullet}{\omega}{}^1{}_{2\varphi} = -\overset{\bullet}{\omega}{}^2{}_{1\varphi} = -r\sin\vartheta\,, \quad \overset{\bullet}{\omega}{}^1{}_{3\vartheta} = -\overset{\bullet}{\omega}{}^3{}_{1\vartheta} = r\,,$$
$$\overset{\bullet}{\omega}{}^1{}_{3\varphi} = -\overset{\bullet}{\omega}{}^3{}_{1\varphi} = -\sqrt{1-r^2}\sin\vartheta\,, \quad \overset{\bullet}{\omega}{}^2{}_{3r} = -\overset{\bullet}{\omega}{}^3{}_{2r} = -\frac{1}{\sqrt{1-r^2}}\,, \quad \overset{\bullet}{\omega}{}^2{}_{3\varphi} = -\overset{\bullet}{\omega}{}^3{}_{2\varphi} = -\cos\vartheta\,.$$

The non-flat case: $k = 1$ Friedmann equations

- Friedmann equations:

$$f + 12 f_T H^2 = 2\kappa^2 \rho \,,$$
$$f - 48 f_{TT} \left(\dot{H} + \frac{1}{a^2}\right) H^2 + 12 f_T H^2 + 4 f_T \left(\dot{H} - \frac{1}{a^2}\right) = -2\kappa^2 p \,.$$

The non-flat case: $k = 1$ Friedmann equations

- Friedmann equations:

$$f + 12 f_T H^2 = 2\kappa^2 \rho\,,$$

$$f - 48 f_{TT}\left(\dot{H} + \frac{1}{a^2}\right) H^2 + 12 f_T H^2 + 4 f_T \left(\dot{H} - \frac{1}{a^2}\right) = -2\kappa^2 p\,.$$

- Constraint equation is essentially unchanged compared to $k = 0$.

The non-flat case: $k = 1$ Friedmann equations

- Friedmann equations:

$$f + 12 f_T H^2 = 2\kappa^2 \rho \,,$$
$$f - 48 f_{TT} \left(\dot{H} + \frac{1}{a^2}\right) H^2 + 12 f_T H^2 + 4 f_T \left(\dot{H} - \frac{1}{a^2}\right) = -2\kappa^2 p \,.$$

- Constraint equation is essentially unchanged compared to $k = 0$.
- Dynamical equation now also depends on the scale factor.

$\Rightarrow$ Additional dimension in dynamical systems analysis.

The non-flat case 2: $k = -1$ cosmology

- Ansatz for $k = -1$ tetrad:

$$\theta^a{}_\mu = \text{diag}\left(1, \frac{a(t)}{\sqrt{1+r^2}}, a(t)r, a(t)r\sin\vartheta\right).$$

The non-flat case 2: $k = -1$ cosmology

- Ansatz for $k = -1$ tetrad:

$$\theta^a{}_\mu = \operatorname{diag}\left(1, \frac{a(t)}{\sqrt{1+r^2}}, a(t)r, a(t)r\sin\vartheta\right).$$

- Resulting FLRW metric:

$$g_{\mu\nu}dx^\mu dx^\nu = dt^2 - a^2(t)\left[\frac{dr^2}{1+r^2} + r^2(d\vartheta^2 + \sin^2\vartheta d\varphi^2)\right].$$

The non-flat case 2: $k = -1$ cosmology

- Ansatz for $k = -1$ tetrad:

$$\theta^a{}_\mu = \operatorname{diag}\left(1, \frac{a(t)}{\sqrt{1+r^2}}, a(t)r, a(t)r\sin\vartheta\right).$$

- Resulting FLRW metric:

$$g_{\mu\nu}dx^\mu dx^\nu = dt^2 - a^2(t)\left[\frac{dr^2}{1+r^2} + r^2(d\vartheta^2 + \sin^2\vartheta d\varphi^2)\right].$$

- Solve antisymmetric part of the field equations using non-vanishing spin connection [MH, Järv, Ualikhanova '18]:

$$\overset{\bullet}{\omega}{}^0{}_{1r} = \overset{\bullet}{\omega}{}^1{}_{0r} = \frac{1}{\sqrt{1+r^2}}, \quad \overset{\bullet}{\omega}{}^0{}_{2\vartheta} = \overset{\bullet}{\omega}{}^2{}_{0\vartheta} = r, \quad \overset{\bullet}{\omega}{}^0{}_{3\varphi} = \overset{\bullet}{\omega}{}^3{}_{0\varphi} = r\sin\vartheta,$$

$$\overset{\bullet}{\omega}{}^1{}_{2\vartheta} = -\overset{\bullet}{\omega}{}^2{}_{1\vartheta} = -\sqrt{1+r^2}, \quad \overset{\bullet}{\omega}{}^1{}_{3\varphi} = -\overset{\bullet}{\omega}{}^3{}_{1\varphi} = -\sqrt{1+r^2}\sin\vartheta, \quad \overset{\bullet}{\omega}{}^2{}_{3\varphi} = -\overset{\bullet}{\omega}{}^3{}_{2\varphi} = -\cos\vartheta.$$

The non-flat case 2: $k = -1$ cosmology

- Ansatz for $k = -1$ tetrad:

$$\theta^a{}_\mu = \operatorname{diag}\left(1, \frac{a(t)}{\sqrt{1+r^2}}, a(t)r, a(t)r\sin\vartheta\right).$$

- Resulting FLRW metric:

$$g_{\mu\nu}dx^\mu dx^\nu = dt^2 - a^2(t)\left[\frac{dr^2}{1+r^2} + r^2(d\vartheta^2 + \sin^2\vartheta d\varphi^2)\right].$$

- Alternative complex choice of the spin connection [Capozziello, Luongo, Richard Pincak, Ravanpak '18]:

$$\overset{\bullet}{\omega}{}^1{}_{2\vartheta} = -\overset{\bullet}{\omega}{}^2{}_{1\vartheta} = -\sqrt{1-r^2}\,, \quad \overset{\bullet}{\omega}{}^1{}_{2\varphi} = -\overset{\bullet}{\omega}{}^2{}_{1\varphi} = -ir\sin\vartheta\,, \quad \overset{\bullet}{\omega}{}^1{}_{3\vartheta} = -\overset{\bullet}{\omega}{}^3{}_{1\vartheta} = ir\,,$$

$$\overset{\bullet}{\omega}{}^1{}_{3\varphi} = -\overset{\bullet}{\omega}{}^3{}_{1\varphi} = -\sqrt{1-r^2}\sin\vartheta\,, \quad \overset{\bullet}{\omega}{}^2{}_{3r} = -\overset{\bullet}{\omega}{}^3{}_{2r} = -\frac{i}{\sqrt{1-r^2}}\,, \quad \overset{\bullet}{\omega}{}^2{}_{3\varphi} = -\overset{\bullet}{\omega}{}^3{}_{2\varphi} = -\cos\vartheta\,.$$

Trouble ahead? Non-unique evolution in $k = -1$ cosmology

- Friedmann equations derived using real spin connection [MH, Järv, Ualikhanova '18]:

$$f + 12f_T H^2 = 2\kappa^2\rho\,,$$

$$f - 48f_{TT}\left(\dot{H} + \frac{H}{a}\right)\left(H - \frac{1}{a}\right)^2 + 12f_T H\left(H - \frac{1}{a}\right) + 4f_T\left(\dot{H} + \frac{1}{a^2}\right) = -2\kappa^2 p\,.$$

Trouble ahead? Non-unique evolution in $k = -1$ cosmology

- Friedmann equations derived using real spin connection [MH, Järv, Ualikhanova '18]:

$$f + 12 f_T H^2 = 2\kappa^2 \rho\,,$$

$$f - 48 f_{TT} \left(\dot{H} + \frac{H}{a}\right)\left(H - \frac{1}{a}\right)^2 + 12 f_T H \left(H - \frac{1}{a}\right) + 4 f_T \left(\dot{H} + \frac{1}{a^2}\right) = -2\kappa^2 p\,.$$

- Friedmann equations derived using complex spin connection [Capozziello, Luongo, Richard Pincak, Ravanpak '18]:

$$f + 12 f_T H^2 = 2\kappa^2 \rho\,,$$

$$f - 48 f_{TT} \left(\dot{H} - \frac{1}{a^2}\right) H^2 + 12 f_T H^2 + 4 f_T \left(\dot{H} + \frac{1}{a^2}\right) = -2\kappa^2 p\,.$$

Trouble ahead? Non-unique evolution in $k = -1$ cosmology

- Friedmann equations derived using real spin connection [MH, Järv, Ualikhanova '18]:

$$f + 12 f_T H^2 = 2\kappa^2 \rho \,,$$
$$f - 48 f_{TT} \left(\dot{H} + \frac{H}{a}\right)\left(H - \frac{1}{a}\right)^2 + 12 f_T H \left(H - \frac{1}{a}\right) + 4 f_T \left(\dot{H} + \frac{1}{a^2}\right) = -2\kappa^2 p \,.$$

- Friedmann equations derived using complex spin connection [Capozziello, Luongo, Richard Pincak, Ravanpak '18]:

$$f + 12 f_T H^2 = 2\kappa^2 \rho \,,$$
$$f - 48 f_{TT} \left(\dot{H} - \frac{1}{a^2}\right) H^2 + 12 f_T H^2 + 4 f_T \left(\dot{H} + \frac{1}{a^2}\right) = -2\kappa^2 p \,.$$

- Different field equations depending on choice of the (unobservable) spin connection.
- Evolution of the Universe depends on a gauge variable?

The next step: scalar-torsion gravity action and field equations

- Add scalar fields $\phi = (\phi^A)$ to the set of dynamical variables.

The next step: scalar-torsion gravity action and field equations

- Add scalar fields $\phi = (\phi^A)$ to the set of dynamical variables.
- Example for gravitational action without derivative couplings [MH, L. Järv, U. Ualikhanova '18]:

$$S = \frac{1}{2\kappa^2} \int_M \left[f(T, \phi) + Z_{AB}(\phi) g^{\mu\nu} \phi^A_{,\mu} \phi^B_{,\nu} \right] \theta d^4x + S_m[\theta^a, \chi^I] \,.$$

The next step: scalar-torsion gravity action and field equations

- Add scalar fields $\phi = (\phi^A)$ to the set of dynamical variables.
- Example for gravitational action without derivative couplings [MH, L. Järv, U. Ualikhanova '18]:

$$S = \frac{1}{2\kappa^2}\int_M \left[f(T,\phi) + Z_{AB}(\phi)g^{\mu\nu}\phi^A_{,\mu}\phi^B_{,\nu}\right]\theta d^4x + S_m[\theta^a,\chi^I]\,.$$

- Field equations:
  - Symmetric part of the tetrad field equations:

$$\frac{1}{2}fg_{\mu\nu} + \overset{\circ}{\nabla}_\rho\left(f_T S_{(\mu\nu)}{}^\rho\right) - \frac{1}{2}f_T S_{(\mu}{}^{\rho\sigma}T_{\nu)\rho\sigma} - Z_{AB}\phi^A_{,\mu}\phi^B_{,\nu} + \frac{1}{2}Z_{AB}\phi^A_{,\rho}\phi^B_{,\sigma}g^{\rho\sigma}g_{\mu\nu} = \kappa^2\Theta_{\mu\nu}\,,$$

  - Antisymmetric part of the tetrad field equations = connection equations:

$$\partial_{[\rho}f_T T^\rho{}_{\mu\nu]} = 0\,.$$

  - Scalar field equation:

$$f_{\phi^A} - \left(2Z_{AB,\phi^C} - Z_{BC,\phi^A}\right)g^{\mu\nu}\phi^B_{,\mu}\phi^C_{,\nu} - 2Z_{AB}\overset{\circ}{\Box}\phi^B = 0\,.$$

- Richer cosmology, can be further generalized [MH '18], [MH, Pfeifer '18] & [MH '18].

A more general torsion scalar: new general relativity

- Action depends on three parameters c_i [Hayashi, Shirafuji '79]:

$$S = \frac{1}{2\kappa^2}\int_M \left(c_1 T^{\mu\nu\rho}T_{\mu\nu\rho} + c_2 T^{\mu\nu\rho}T_{\rho\nu\mu} + c_3 T^{\mu}{}_{\mu\rho}T_{\nu}{}^{\nu\rho}\right)\theta\, d^4x + S_m[\theta^a, \chi^I]\,.$$

A more general torsion scalar: new general relativity

- Action depends on three parameters c_i [Hayashi, Shirafuji '79]:

$$S = \frac{1}{2\kappa^2} \int_M \left(c_1 T^{\mu\nu\rho} T_{\mu\nu\rho} + c_2 T^{\mu\nu\rho} T_{\rho\nu\mu} + c_3 T^{\mu}{}_{\mu\rho} T_{\nu}{}^{\nu\rho} \right) \theta \, d^4x + S_m[\theta^a, \chi^I] \, .$$

- Reduces to TEGR for $c_1 = \frac{1}{4}$, $c_2 = \frac{1}{2}$, $c_3 = -1$.

A more general torsion scalar: new general relativity

- Action depends on three parameters c_i [Hayashi, Shirafuji '79]:

$$S = \frac{1}{2\kappa^2}\int_M \left(c_1 T^{\mu\nu\rho}T_{\mu\nu\rho} + c_2 T^{\mu\nu\rho}T_{\rho\nu\mu} + c_3 T^{\mu}{}_{\mu\rho}T_{\nu}{}^{\nu\rho}\right)\theta\, d^4x + S_m[\theta^a, \chi^I]\,.$$

- Reduces to TEGR for $c_1 = \frac{1}{4}$, $c_2 = \frac{1}{2}$, $c_3 = -1$.
- Cosmology independent of parameters.

A more general torsion scalar: new general relativity

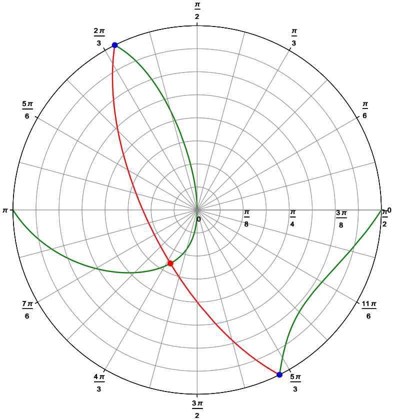
- Action depends on three parameters c_i [Hayashi, Shirafuji '79]:

$$S = \frac{1}{2\kappa^2}\int_M \left(c_1 T^{\mu\nu\rho}T_{\mu\nu\rho} + c_2 T^{\mu\nu\rho}T_{\rho\nu\mu} + c_3 T^{\mu}{}_{\mu\rho}T_{\nu}{}^{\nu\rho}\right)\theta\, d^4x + S_m[\theta^a, \chi^I]\,.$$

- Reduces to TEGR for $c_1 = \frac{1}{4}$, $c_2 = \frac{1}{2}$, $c_3 = -1$.
- Cosmology independent of parameters.
- Distinguishable by gravitational waves:

  [MH, Krššák, Pfeifer, Ualikhanova '18]

  - II_6 - 6 polarizations.
  - III_5 - 5 polarizations.
  - N_3 - 3 polarizations.
  - N_2 - 2 polarizations.



Outline

Gravity in the symmetric teleparallel language

Curvature gravity	Non-metricity gravity
Fundamental fields	
Metric $g_{\mu\nu}$	Metric $g_{\mu\nu}$ Connection $\overset{\times}{\Gamma}{}^{\mu}{}_{\nu\rho}$
Constraints	
-	$\overset{\times}{\Gamma}{}^{\rho}{}_{[\nu\mu]} = 0$ $\partial_{[\rho}\overset{\times}{\Gamma}{}^{\mu}{}_{\lvert\nu\rvert\sigma]} + \overset{\times}{\Gamma}{}^{\mu}{}_{\tau[\rho}\overset{\times}{\Gamma}{}^{\tau}{}_{\lvert\nu\rvert\sigma]} = 0$
Derived quantities	
Connection $\overset{\circ}{\Gamma}{}^{\rho}{}_{\mu\nu} = \frac{1}{2}g^{\rho\sigma}\left(g_{\mu\sigma,\nu} + g_{\nu\sigma,\mu} - g_{\mu\nu,\sigma}\right)$	-
Quantity mediating gravity	
Curvature $\overset{\circ}{R}{}^{\mu}{}_{\nu\rho\sigma} = 2\left(\partial_{[\rho}\overset{\circ}{\Gamma}{}^{\mu}{}_{\lvert\nu\rvert\sigma]} + \overset{\circ}{\Gamma}{}^{\mu}{}_{\tau[\rho}\overset{\circ}{\Gamma}{}^{\tau}{}_{\lvert\nu\rvert\sigma]}\right)$	Non-metricity $\overset{\times}{Q}_{\rho\mu\nu} = \overset{\times}{\nabla}_{\rho}g_{\mu\nu}$
Vanishing quantities	
$\overset{\circ}{Q}_{\rho\mu\nu} = \overset{\circ}{\nabla}_{\rho}g_{\mu\nu} = 0$	$\overset{\times}{R}{}^{\mu}{}_{\nu\rho\sigma} = 2\left(\partial_{[\rho}\overset{\times}{\Gamma}{}^{\mu}{}_{\lvert\nu\rvert\sigma]} + \overset{\times}{\Gamma}{}^{\mu}{}_{\tau[\rho}\overset{\times}{\Gamma}{}^{\tau}{}_{\lvert\nu\rvert\sigma]}\right) = 0$

Modified teleparallel gravity: $f(Q)$ theory action and field equations

- Gravitational action [Beltran Jimenez, Heisenberg, Koivisto '17]:

$$S = \frac{1}{2\kappa^2}\int_M f(Q)\sqrt{-g}\,d^4x + S_m[g_{\mu\nu}, \chi^I]\,.$$

Modified teleparallel gravity: $f(Q)$ theory action and field equations

- Gravitational action [Beltran Jimenez, Heisenberg, Koivisto '17]:

$$S = \frac{1}{2\kappa^2}\int_M f(Q)\sqrt{-g}\, d^4x + S_m[g_{\mu\nu}, \chi^I]\,.$$

- Field equations ... are a bit lengthy, and therefore not shown here. But we remark, that the connection equations are simply the divergence of the metric equations, and are thus equivalent to the Bianchi identities.

Modified teleparallel gravity: $f(Q)$ theory action and field equations

- Gravitational action [Beltran Jimenez, Heisenberg, Koivisto '17]:

$$S = \frac{1}{2\kappa^2} \int_M f(Q) \sqrt{-g}\, d^4x + S_m[g_{\mu\nu}, \chi^I] \,.$$

- Field equations ... are a bit lengthy, and therefore not shown here. But we remark, that the connection equations are simply the divergence of the metric equations, and are thus equivalent to the Bianchi identities.
- Terms appearing in the action and field equations:
  - Superpotential: $P^\alpha{}_{\mu\nu} = -\frac{1}{4}Q^\alpha{}_{\mu\nu} + \frac{1}{2}Q_{(\mu}{}^\alpha{}_{\nu)} + \frac{1}{4}Q^\alpha g_{\mu\nu} - \frac{1}{4}\left(\tilde{Q}^\alpha g_{\mu\nu} + \delta^\alpha_{(\mu} Q_{\nu)}\right)$.

Modified teleparallel gravity: $f(Q)$ theory action and field equations

- Gravitational action [Beltran Jimenez, Heisenberg, Koivisto '17]:

$$S = \frac{1}{2\kappa^2} \int_M f(Q) \sqrt{-g}\, d^4x + S_m[g_{\mu\nu}, \chi^I] \,.$$

- Field equations ... are a bit lengthy, and therefore not shown here. But we remark, that the connection equations are simply the divergence of the metric equations, and are thus equivalent to the Bianchi identities.
- Terms appearing in the action and field equations:
  - Superpotential: $P^{\alpha}{}_{\mu\nu} = -\frac{1}{4} Q^{\alpha}{}_{\mu\nu} + \frac{1}{2} Q_{(\mu}{}^{\alpha}{}_{\nu)} + \frac{1}{4} Q^{\alpha} g_{\mu\nu} - \frac{1}{4} \left(\tilde{Q}^{\alpha} g_{\mu\nu} + \delta^{\alpha}_{(\mu} Q_{\nu)} \right)$.
  - Non-metricity scalar: $Q = Q^{\rho}{}_{\mu\nu} P_{\rho}{}^{\mu\nu}$ and vectors $Q_{\mu} = Q^{\nu}{}_{\nu\mu}$ & $\tilde{Q}_{\mu} = Q_{\mu\nu}{}^{\nu}$.

Modified teleparallel gravity: $f(Q)$ theory action and field equations

- Gravitational action [Beltran Jimenez, Heisenberg, Koivisto '17]:

$$S = \frac{1}{2\kappa^2}\int_M f(Q)\sqrt{-g}\,d^4x + S_m[g_{\mu\nu}, \chi^I]\,.$$

- Field equations . . . are a bit lengthy, and therefore not shown here. But we remark, that the connection equations are simply the divergence of the metric equations, and are thus equivalent to the Bianchi identities.
- Terms appearing in the action and field equations:
  - Superpotential: $P^\alpha{}_{\mu\nu} = -\frac{1}{4}Q^\alpha{}_{\mu\nu} + \frac{1}{2}Q_{(\mu}{}^\alpha{}_{\nu)} + \frac{1}{4}Q^\alpha g_{\mu\nu} - \frac{1}{4}\left(\tilde{Q}^\alpha g_{\mu\nu} + \delta^\alpha_{(\mu}Q_{\nu)}\right)$.
  - Non-metricity scalar: $Q = Q^\rho{}_{\mu\nu}P_\rho{}^{\mu\nu}$ and vectors $Q_\mu = Q^\nu{}_{\nu\mu}$ & $\tilde{Q}_\mu = Q_{\mu\nu}{}^\nu$.
  - Energy-momentum tensor $\Theta_{\mu\nu}$ derived from the matter part S_m of the action.

$f(Q)$ cosmology and field equations

- Choose *coincident gauge* $\overset{\times}{\Gamma}^{\mu}{}_{\nu\rho} = 0$ and $k = 0$ FLRW metric [Beltran Jimenez, Heisenberg, Koivisto '17]

$$g_{\mu\nu}dx^{\mu}dx^{\nu} = dt^2 - a^2(t)\delta_{ij}dx^i dx^j \,.$$

$f(Q)$ cosmology and field equations

- Choose *coincident gauge* $\overset{\times}{\Gamma}{}^{\mu}{}_{\nu\rho} = 0$ and $k = 0$ FLRW metric [Beltran Jimenez, Heisenberg, Koivisto '17]

$$g_{\mu\nu}dx^{\mu}dx^{\nu} = dt^2 - a^2(t)\delta_{ij}dx^i dx^j \,.$$

- Cosmological field equations:

$$\begin{aligned} 12H^2 f_Q + f &= 2\kappa^2\rho \,, \\ 48H^2\dot{H}f_{QQ} - (12H^2 + 4\dot{H})f_Q - f &= 2\kappa^2 p \,, \end{aligned}$$

$f(Q)$ cosmology and field equations

- Choose *coincident gauge* $\overset{\times}{\Gamma}^{\mu}{}_{\nu\rho} = 0$ and $k = 0$ FLRW metric [Beltran Jimenez, Heisenberg, Koivisto '17]

$$g_{\mu\nu}dx^{\mu}dx^{\nu} = dt^2 - a^2(t)\delta_{ij}dx^i dx^j\,.$$

- Cosmological field equations:

$$12H^2 f_Q + f = 2\kappa^2\rho\,,$$
$$48H^2\dot{H}f_{QQ} - (12H^2 + 4\dot{H})f_Q - f = 2\kappa^2 p\,,$$

- Cosmological dynamics essentially equivalent to $f(T)$ cosmology.

Scalar-torsion gravity and cosmology

- Action involves in addition also scalar field ϕ [Järv, Rünkla, Saal, Vilson '18]:

$$S = \frac{1}{2\kappa^2}\int_M [\mathcal{A}(\phi)Q - \mathcal{B}(\phi)g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} - 2\mathcal{V}(\phi)]\sqrt{-g}\,d^4x + S_m[g_{\mu\nu},\chi^I]\,.$$

Scalar-torsion gravity and cosmology

- Action involves in addition also scalar field ϕ [Järv, Rünkla, Saal, Vilson '18]:

$$S = \frac{1}{2\kappa^2} \int_M [\mathcal{A}(\phi) Q - \mathcal{B}(\phi) g^{\mu\nu} \phi_{,\mu} \phi_{,\nu} - 2\mathcal{V}(\phi)] \sqrt{-g}\, d^4x + S_m[g_{\mu\nu}, \chi^I] .$$

- Cosmological dynamics for $k = 0$ FLRW metric:

$$H^2 = \frac{1}{3\mathcal{A}} \left(\kappa^2 \rho + \frac{1}{2} \mathcal{B} \dot{\phi}^2 + \mathcal{V} \right) ,$$

$$2\dot{H} + 3H^2 = \frac{1}{\mathcal{A}} \left(-2\mathcal{A}' H \dot{\phi} - \frac{1}{2} \mathcal{B} \dot{\phi}^2 + \mathcal{V} - \kappa^2 p \right)$$

$$0 = \mathcal{B} \ddot{\phi} + \left(3\mathcal{B} H + \frac{1}{2} \mathcal{B}' \dot{\phi} \right) \dot{\phi} + \mathcal{V}' + 3\mathcal{A}' H^2 .$$

Scalar-torsion gravity and cosmology

- Action involves in addition also scalar field ϕ [Järv, Rünkla, Saal, Vilson '18]:

$$S = \frac{1}{2\kappa^2}\int_M [\mathcal{A}(\phi)Q - \mathcal{B}(\phi)g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} - 2\mathcal{V}(\phi)]\sqrt{-g}\,d^4x + S_m[g_{\mu\nu},\chi^I]\,.$$

- Cosmological dynamics for $k = 0$ FLRW metric:

$$H^2 = \frac{1}{3\mathcal{A}}\left(\kappa^2\rho + \frac{1}{2}\mathcal{B}\dot{\phi}^2 + \mathcal{V}\right),$$

$$2\dot{H} + 3H^2 = \frac{1}{\mathcal{A}}\left(-2\mathcal{A}'H\dot{\phi} - \frac{1}{2}\mathcal{B}\dot{\phi}^2 + \mathcal{V} - \kappa^2 p\right)$$

$$0 = \mathcal{B}\ddot{\phi} + \left(3\mathcal{B}H + \frac{1}{2}\mathcal{B}'\dot{\phi}\right)\dot{\phi} + \mathcal{V}' + 3\mathcal{A}'H^2\,.$$

- Rich cosmology that deserves further studies (dynamical system).

A more general non-metricity scalar: newer general relativity

- Action depends on five parameters c_i [Beltran Jimenez, Heisenberg, Koivisto '17]:

$$S = \frac{1}{2\kappa^2}\int_M \left(c_1 Q^{\rho\mu\nu}Q_{\rho\mu\nu} + c_2 Q^{\rho\mu\nu}Q_{\nu\mu\rho} + c_3 Q^{\mu}Q_{\mu} + c_4 \tilde{Q}^{\mu}\tilde{Q}_{\mu} + c_5 \tilde{Q}^{\mu}Q_{\mu}\right)\sqrt{-g}\,d^4x + S_m[g_{\mu\nu},\chi^I]$$

A more general non-metricity scalar: newer general relativity

- Action depends on five parameters c_i [Beltran Jimenez, Heisenberg, Koivisto '17]:

$$S = \frac{1}{2\kappa^2}\int_M \left(c_1 Q^{\rho\mu\nu}Q_{\rho\mu\nu} + c_2 Q^{\rho\mu\nu}Q_{\nu\mu\rho} + c_3 Q^{\mu}Q_{\mu} + c_4 \tilde{Q}^{\mu}\tilde{Q}_{\mu} + c_5 \tilde{Q}^{\mu}Q_{\mu}\right)\sqrt{-g}\,d^4x + S_m[g_{\mu\nu}, \chi^I]$$

- Reduces to STEGR for
  $c_1 = -\frac{1}{4}$, $c_2 = \frac{1}{2}$, $c_3 = \frac{1}{4}$, $c_4 = 0$, $c_5 = -\frac{1}{2}$.

A more general non-metricity scalar: newer general relativity

- Action depends on five parameters c_i [Beltran Jimenez, Heisenberg, Koivisto '17]:

$$S = \frac{1}{2\kappa^2}\int_M \left(c_1 Q^{\rho\mu\nu}Q_{\rho\mu\nu} + c_2 Q^{\rho\mu\nu}Q_{\nu\mu\rho} + c_3 Q^{\mu}Q_{\mu} + c_4 \tilde{Q}^{\mu}\tilde{Q}_{\mu} + c_5 \tilde{Q}^{\mu}Q_{\mu}\right)\sqrt{-g}\,d^4x + S_m[g_{\mu\nu},\chi^I]$$

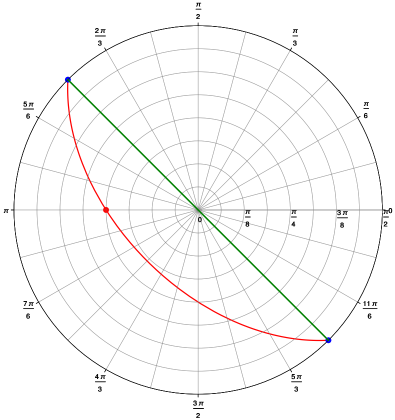
- Reduces to STEGR for
  $c_1 = -\frac{1}{4}$, $c_2 = \frac{1}{2}$, $c_3 = \frac{1}{4}$, $c_4 = 0$, $c_5 = -\frac{1}{2}$.
- Cosmology independent of parameters.

A more general non-metricity scalar: newer general relativity

- Action depends on five parameters c_i [Beltran Jimenez, Heisenberg, Koivisto '17]:

$$S = \frac{1}{2\kappa^2}\int_M \left(c_1 Q^{\rho\mu\nu}Q_{\rho\mu\nu} + c_2 Q^{\rho\mu\nu}Q_{\nu\mu\rho} + c_3 Q^{\mu}Q_{\mu} + c_4 \tilde{Q}^{\mu}\tilde{Q}_{\mu} + c_5 \tilde{Q}^{\mu}Q_{\mu}\right)\sqrt{-g}\,d^4x + S_m[g_{\mu\nu},\chi^I]$$

- Reduces to STEGR for
  $c_1 = -\frac{1}{4},\ c_2 = \frac{1}{2},\ c_3 = \frac{1}{4},\ c_4 = 0,\ c_5 = -\frac{1}{2}.$
- Cosmology independent of parameters.
- Distinguishable by gravitational waves:
  [MH, Pfeifer, Said, Ualikhanova '18]
  - II_6 - 6 polarizations. (blue)
  - III_5 - 5 polarizations. (green)
  - N_3 - 3 polarizations. (white)
  - N_2 - 2 polarizations. (red)



Outline

Conclusion

- Summary:
  - Teleparallel and symmetric teleparallel gravity use geometries without curvature.
  - Gravity is mediated by torsion in teleparallel gravity.
  - Gravity is mediated by non-metricity in symmetric teleparallel gravity.
  - Rich cosmology in $f(T)$ and $f(Q)$ theories.
  - Even richer cosmology when adding scalar fields, different from scalar-curvature.
  - Possible ambiguity in cosmological evolution for certain cases.
  - Certain theories can be distinguished using gravitational waves.

Conclusion

- Summary:
  - Teleparallel and symmetric teleparallel gravity use geometries without curvature.
  - Gravity is mediated by torsion in teleparallel gravity.
  - Gravity is mediated by non-metricity in symmetric teleparallel gravity.
  - Rich cosmology in $f(T)$ and $f(Q)$ theories.
  - Even richer cosmology when adding scalar fields, different from scalar-curvature.
  - Possible ambiguity in cosmological evolution for certain cases.
  - Certain theories can be distinguished using gravitational waves.
- Outlook:
  - Enhance analysis of cosmology by, e.g., cosmological perturbations.
  - Resolve ambiguity in cosmological solutions and evolution.
  - Obtain constraints on shown theories, chart the landscape of parameters.
  - Describe the Universe as a whole in teleparallel gravity!