“Cosmological” tetrads and spin connections in teleparallel gravity

How to solve the antisymmetric part of the field equations by using symmetry

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  - Accelerating phases in the history of the Universe?
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  - Simple class of teleparallel theories beyond general relativity.
  - Contains $f(T)$ gravity [Bengochea, Ferraro '09].
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  - ↯ Cosmological symmetry determines tetrad only up to local Lorentz transformation.
  - ↯ Need to solve antisymmetric part of field equations for the spin connection.
  - Use presence of cosmological symmetry to find particular solutions?

Ingredients of scalar-torsion gravity

- Fundamental fields:
  - Coframe field $\theta^a = \theta^a{}_\mu \mathrm{d}x^\mu$.
  - Flat spin connection $\overset{\bullet}{\omega}{}^a{}_b = \overset{\bullet}{\omega}{}^a{}_{b\mu} \mathrm{d}x^\mu$.
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- Derived quantities:
  - Frame field $e_a = e_a{}^\mu \partial_\mu$ with $\iota_{e_a}\theta^b = \delta^b_a$.
  - Metric $g_{\mu\nu} = \eta_{ab}\theta^a{}_\mu \theta^b{}_\nu$.
  - Volume form $\theta \mathrm{d}^4x = \theta^0 \wedge \theta^1 \wedge \theta^2 \wedge \theta^3$.
  - Levi-Civita connection

$$\overset{\circ}{\omega}_{ab} = -\frac{1}{2}(\iota_{e_b}\iota_{e_c}\mathrm{d}\theta_a + \iota_{e_c}\iota_{e_a}\mathrm{d}\theta_b - \iota_{e_a}\iota_{e_b}\mathrm{d}\theta_c)\theta^c\,.$$

  - Torsion $T^a = \mathrm{d}\theta^a + \overset{\bullet}{\omega}{}^a{}_b \wedge \theta^b$.

Scalar-torsion gravity action and field equations

- Gravitational action [MH, L. Järv, U. Ualikhanova '18]:

$$S = \frac{1}{2\kappa^2}\int_M \left[f(T,\phi) + Z_{AB}(\phi)g^{\mu\nu}\phi^A_{,\mu}\phi^B_{,\nu}\right]\theta d^4x + S_m[\theta^a,\chi^I]\,.$$

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- Field equations:
  - Symmetric part of the tetrad field equations:

$$\frac{1}{2}fg_{\mu\nu} + \overset{\circ}{\nabla}_\rho\left(f_T S_{(\mu\nu)}{}^\rho\right) - \frac{1}{2}f_T S_{(\mu}{}^{\rho\sigma}T_{\nu)\rho\sigma} - Z_{AB}\phi^A_{,\mu}\phi^B_{,\nu} + \frac{1}{2}Z_{AB}\phi^A_{,\rho}\phi^B_{,\sigma}g^{\rho\sigma}g_{\mu\nu} = \kappa^2\Theta_{\mu\nu}\,,$$

  - Antisymmetric part of the tetrad field equations:

$$\partial_{[\rho}f_T T^\rho{}_{\mu\nu]} = 0\,.$$

  - Scalar field equation:

$$f_{\phi^A} - \left(2Z_{AB,\phi^C} - Z_{BC,\phi^A}\right)g^{\mu\nu}\phi^B_{,\mu}\phi^C_{,\nu} - 2Z_{AB}\overset{\circ}{\square}\phi^B = 0\,.$$

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- Solutions to the antisymmetric part of the equations?

Four ways to solve $\iota_{V_{ab}} \mathrm{d} f_T = 0$

- Different possibilities to solve this equation:

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- Consider group action on M with orbits of codimension 1.
- Choose geometry to be symmetric under this group action.
- Determine vector fields V_{ab} such that they are tangent to orbits..

Symmetry of the geometry

- Diffeomorphisms generated by vector field ξ.
- Invariance of spacetime geometry:
  - Metric:

$$0 = (\mathcal{L}_\xi g)_{\mu\nu} = \xi^\rho \partial_\rho g_{\mu\nu} + \partial_\mu \xi^\rho g_{\rho\nu} + \partial_\nu \xi^\rho g_{\mu\rho} \,.$$

  - Connection:

$$0 = (\mathcal{L}_\xi \Gamma)^\mu{}_{\nu\rho} = \xi^\sigma \partial_\sigma \Gamma^\mu{}_{\nu\rho} - \partial_\sigma \xi^\mu \Gamma^\sigma{}_{\nu\rho} + \partial_\nu \xi^\sigma \Gamma^\mu{}_{\sigma\rho} + \partial_\rho \xi^\sigma \Gamma^\mu{}_{\nu\sigma} + \partial_\nu \partial_\rho \xi^\mu \,.$$

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- Satisfied if and only if $\exists\lambda : M \to \mathfrak{so}(1,3)$ such that [MH '15]

$$(\mathcal{L}_\xi e)^a{}_\mu = -\lambda^a{}_b e^b{}_\mu\,, \quad (\mathcal{L}_\xi\omega)^a{}_{b\mu} = D_\mu\lambda^a{}_b\,.$$

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- Several symmetry generators ξ form Lie algebra $\mathfrak{g} \subset \mathrm{Vect}(M)$.
- Local Lie algebra homomorphism $\boldsymbol{\lambda} : \mathfrak{g} \times M \to \mathfrak{so}(1,3)$.

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A tetrad is called *good tetrad* if it satisfies the equation

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⇒ Lie algebra homomorphism $\boldsymbol{\lambda} : \mathfrak{g} \to \mathfrak{so}(1,3)$ (independent of M).

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- Use local Lorentz transformation to go to arbitrary (e.g., diagonal) gauge.
- Use additional condition also for the scalar fields.

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- Choice of representation fixes tetrad up to $n(t), a(t)$:

$$e^a{}_\mu = \begin{pmatrix} n(t) & 0 & 0 & 0 \\ 0 & a(t)\sin\theta\cos\phi & a(t)r\cos\theta\cos\phi & -a(t)r\sin\theta\sin\phi \\ 0 & a(t)\sin\theta\sin\phi & a(t)r\cos\theta\sin\phi & a(t)r\sin\theta\cos\phi \\ 0 & a(t)\cos\theta & -a(t)r\sin\theta & 0 \end{pmatrix}.$$

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Cosmological field equations for flat FLRW

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- Remaining field equations:
  - Tetrad equations:

$$\frac{1}{2}f + 6f_T H^2 - \frac{1}{2}Z\dot{\phi}^2 = \kappa^2\rho\,,$$

$$\frac{1}{2}f + 2f_{T\phi}H\dot{\phi} - 24f_{TT}\dot{H}H^2 + 6f_T H^2 + 2f_T\dot{H} + \frac{1}{2}Z\dot{\phi}^2 = -\kappa^2 p\,,$$

  - Scalar field equation:

$$f_\phi - 2Z\ddot{\phi} - 6ZH\dot{\phi} - Z_\phi\dot{\phi}^2 = 0\,.$$

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- Same cosmological field equations, since $\boldsymbol{\lambda}$ is equivalent representation.

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- Different cosmological field equations, since λ is inequivalent representation.

Conclusion

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- Further reading:

  - MH; *Spacetime and observer space symmetries in the language of Cartan geometry*; J. Math. Phys. **57** (2016) 082502 [arXiv:1505.07809].
  - MH, L. Järv, U. Ualikhanova; *Covariant formulation of scalar-torsion gravity*; Phys. Rev. D **97** (2018) 104011 [arXiv:1801.05786].
  - MH, L. Järv, C. Pfeifer, M. Krššák; *Modified teleparallel theories of gravity in symmetric spacetimes*; to appear.