Gravitational waves in teleparallel theories of gravity

Manuel Hohmann
(with Martin Krššák, Christian Pfeifer, Jackson Levi Said, Ulbossyn Ualikhanova)

Laboratory of Theoretical Physics - Institute of Physics - University of Tartu
Center of Excellence "The Dark Side of the Universe"

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Outline

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Motivation

- Open questions in cosmology and gravity:
  - Accelerating phases in the history of the Universe?
  - Relation between gravity and gauge theories?
  - How to quantize gravity?

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  - Based on tetrad and flat spin connection.
  - Describes gravity as gauge theory of the translation group.
  - First order action, second order field equations.
  - Spin connection as Lorentz gauge degree of freedom.

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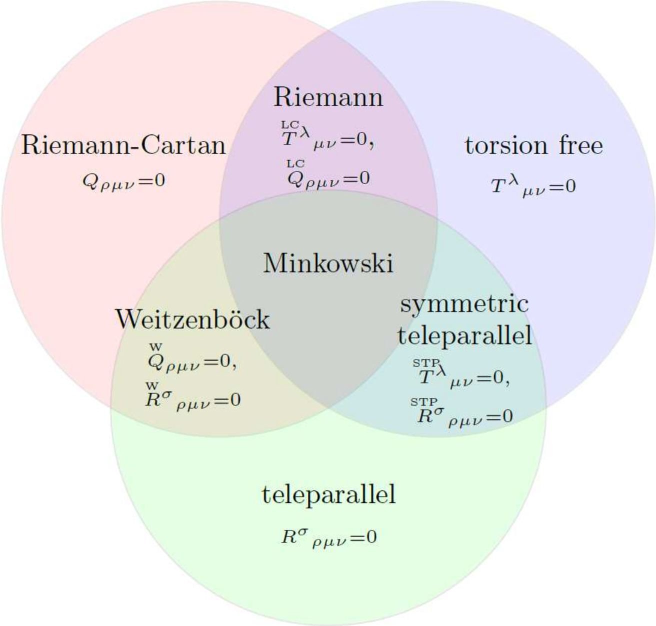
- Gravity formulated as gauge theories.

Overview of geometries



Newman-Penrose formalism

- Complex double null basis of the tangent bundle:

$$l = \partial_t + \partial_z\,, \quad n = \frac{\partial_t - \partial_z}{2}\,, \quad m = \frac{\partial_x + i\partial_y}{\sqrt{2}}\,, \quad \bar{m} = \frac{\partial_x - i\partial_y}{\sqrt{2}}\,.$$

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- Consider plane null wave with $k_\mu = -\omega l_\mu$ and $u = t - z$:

$$h_{\mu\nu} = H_{\mu\nu} e^{ik_\mu x^\mu} = H_{\mu\nu} e^{i\omega u} \,.$$

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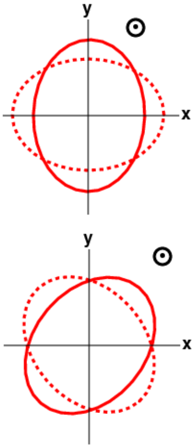
- Riemann tensor determined by “electric” components:

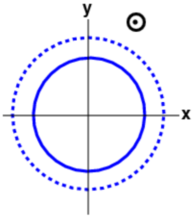
$$\Psi_2 = -\frac{1}{6}R_{nlnl} = \frac{1}{12}\ddot{h}_{ll}\,, \qquad \Psi_3 = -\frac{1}{2}R_{nln\bar{m}} = \frac{1}{4}\ddot{h}_{l\bar{m}}\,,$$

$$\Psi_4 = -R_{n\bar{m}n\bar{m}} = \frac{1}{2}\ddot{h}_{\bar{m}\bar{m}}\,, \qquad \Phi_{22} = -R_{nmn\bar{m}} = \frac{1}{2}\ddot{h}_{m\bar{m}}\,.$$

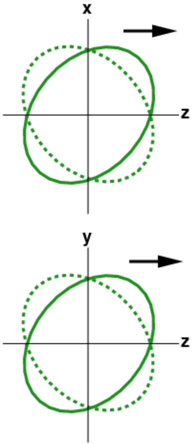
Polarisations of gravitational waves



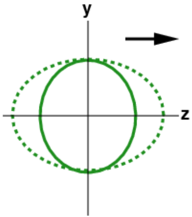
$\Psi_4, \bar{\Psi}_4$



Φ_{22}



$\Psi_3, \bar{\Psi}_3$



Ψ_2

Outline

Field content and geometry

- Fundamental fields in the gravity sector:
  - Coframe field $\theta^a = \theta^a{}_\mu \mathrm{d}x^\mu$.
  - Flat spin connection $\overset{\bullet}{\omega}{}^a{}_b = \overset{\bullet}{\omega}{}^a{}_{b\mu} \mathrm{d}x^\mu$.

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- Derived quantities:
  - Frame field $e_a = e_a{}^\mu \partial_\mu$ with $\iota_{e_a}\theta^b = \delta^b_a$.
  - Metric $g_{\mu\nu} = \eta_{ab}\theta^a{}_\mu \theta^b{}_\nu$.
  - Volume form $\theta \mathrm{d}^4x = \theta^0 \wedge \theta^1 \wedge \theta^2 \wedge \theta^3$.
  - Levi-Civita connection

$$\overset{\circ}{\omega}_{ab} = -\frac{1}{2}(\iota_{e_b}\iota_{e_c}\mathrm{d}\theta_a + \iota_{e_c}\iota_{e_a}\mathrm{d}\theta_b - \iota_{e_a}\iota_{e_b}\mathrm{d}\theta_c)\theta^c \,.$$

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  - Torsion $T^a = \mathrm{d}\theta^a + \overset{\bullet}{\omega}{}^a{}_b \wedge \theta^b$.
- Gauge fixing
  - Perform local Lorentz transformation:

$$\theta'^a = \Lambda^a{}_b\theta^b\,, \quad \overset{\bullet}{\omega}{}'^a{}_b = \Lambda^a{}_c\overset{\bullet}{\omega}{}^c{}_d\Lambda_b{}^d + \Lambda^a{}_c\mathrm{d}\Lambda_b{}^c\,.$$

$\Rightarrow$ Weitzenböck gauge: set $\overset{\bullet}{\omega}{}^a{}_b \equiv 0$.

Most general action and corresponding field equations

- Most general action:

$$S = \frac{1}{2\kappa^2} \int d^4x \, e \left(c_1 T^{\mu\nu\rho} T_{\mu\nu\rho} + c_2 T^{\mu\nu\rho} T_{\rho\nu\mu} + c_3 T^{\mu}{}_{\mu\rho} T_{\nu}{}^{\nu\rho} \right) .$$

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- Linearized vacuum field equations:

$$\partial_\sigma \left(F^{\mu\rho\sigma} + B^{\mu\rho\sigma} \right) = 0 .$$

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- Field tensors:
  - Symmetric perturbation part:

$$F^{\mu\rho\sigma} = (2c_1 + c_2)\left(\partial^\sigma \phi^{\mu\rho} - \partial^\rho \phi^{\mu\sigma} \right) + c_3 \left[(\partial^\sigma \phi^\alpha{}_\alpha - \partial_\alpha \phi^{\alpha\sigma})\eta^{\mu\rho} - (\partial^\rho \phi^\alpha{}_\alpha - \partial_\alpha \phi^{\alpha\rho})\eta^{\mu\sigma} \right] .$$

  - Antisymmetric perturbation part:

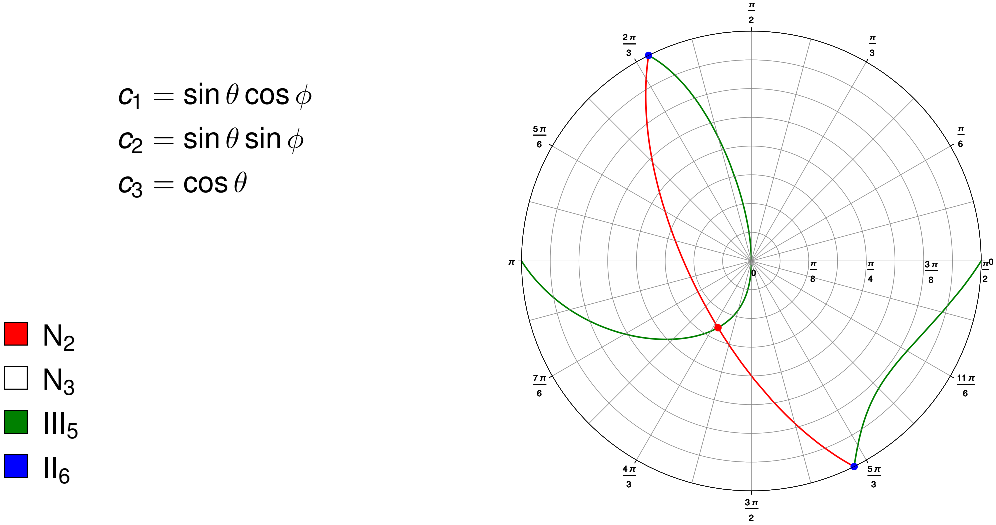
$$B^{\mu\rho\sigma} = (2c_1 - c_2)\left(\partial^\sigma a^{\mu\rho} - \partial^\rho a^{\mu\sigma} \right) + (2c_2 + c_3)\partial^\mu a^{\sigma\rho} \, .$$

Newman-Penrose decomposition

Field equations expressed in Newman-Penrose basis

$$
\begin{aligned}
0 = E_{nn} &= (2c_1 + c_2 + c_3)\partial_n^2\phi_{nl} + 2c_3\phi_{m\bar{m}} + (2c_1 + c_2 + c_3)\partial_n^2 a_{nl}\,,\\
0 = E_{mn} &= (2c_1 + c_2)\partial_n^2\phi_{ml} + (2c_1 - c_2)\partial_n^2 a_{ml}\,,\\
0 = E_{\bar{m}n} &= (2c_1 + c_2)\partial_n^2\phi_{\bar{m}l} + (2c_1 - c_2)\partial_n^2 a_{\bar{m}l}\,,\\
0 = E_{nm} &= -c_3\partial_n^2\phi_{lm} - (2c_2 + c_3)\partial_n^2 a_{lm}\,,\\
0 = E_{n\bar{m}} &= -c_3\partial_n^2\phi_{l\bar{m}} - (2c_2 + c_3)\partial_n^2 a_{l\bar{m}}\\
0 = E_{m\bar{m}} &= -c_3\partial_n^2\phi_{ll}\,,\\
0 = E_{ln} &= (2c_1 + c_2)\partial_n^2\phi_{ll}\,,\,.
\end{aligned}
$$

$$c_1 = \sin\theta\cos\phi$$
$$c_2 = \sin\theta\sin\phi$$
$$c_3 = \cos\theta$$



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  - Volume form $\sqrt{-\det g}\mathrm{d}^4x$.
  - Levi-Civita connection

$$\overset{\circ}{\Gamma}{}^{\rho}{}_{\mu\nu} = \frac{1}{2}g^{\rho\sigma}(\partial_{\mu}g_{\sigma\nu} + \partial_{\nu}g_{\mu\sigma} - \partial_{\sigma}g_{\mu\nu})\,.$$

  - Non-metricity $Q_{\rho\mu\nu} = \overset{\times}{\nabla}_{\rho}g_{\mu\nu}$.

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- Gauge fixing
  - Perform local coordinate transformation:

$$g'_{\mu\nu} = \frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial x^\beta}{\partial x'^\nu}g_{\alpha\beta}\,, \quad \overset{\times}{\Gamma}{}'^{\rho}{}_{\mu\nu} = \frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial x^\beta}{\partial x'^\nu}\frac{\partial x'^\rho}{\partial x^\gamma}\overset{\times}{\Gamma}{}^{\gamma}{}_{\alpha\beta} + \frac{\partial^2 x^\alpha}{\partial x'^\mu \partial x'^\nu}\frac{\partial x'^\rho}{\partial x^\alpha}\,.$$

$\Rightarrow$ Coincident gauge: set $\overset{\times}{\Gamma}{}^{\rho}{}_{\mu\nu} \equiv 0 \Rightarrow Q_{\rho\mu\nu} = \partial_\rho g_{\mu\nu}$.

Most general action and corresponding field equations

- Most general action:

$$S = -\int d^4x \frac{\sqrt{-g}}{2}\left[c_1 Q^{\alpha}{}_{\mu\nu} + c_2 Q_{(\mu}{}^{\alpha}{}_{\nu)} + c_3 Q^{\alpha} g_{\mu\nu} + c_4 \delta^{\alpha}_{(\mu}\tilde{Q}_{\nu)} + \frac{c_5}{2}\left(\tilde{Q}^{\alpha} g_{\mu\nu} + \delta^{\alpha}_{(\mu} Q_{\nu)}\right)\right]$$

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- Linearized field equations:

$$\begin{aligned}0 = {} & 2c_1\eta^{\alpha\sigma}\partial_\alpha\partial_\sigma h_{\mu\nu} + c_2\eta^{\alpha\sigma}\left(\partial_\alpha\partial_\mu h_{\sigma\nu} + \partial_\alpha\partial_\nu h_{\sigma\mu}\right) + 2c_3\eta_{\mu\nu}\eta^{\tau\omega}\eta^{\alpha\sigma}\partial_\alpha\partial_\sigma h_{\tau\omega} \\ & + c_4\eta^{\omega\sigma}(\partial_\mu\partial_\omega h_{\nu\sigma} + \partial_\nu\partial_\omega h_{\mu\sigma}) + c_5\eta_{\mu\nu}\eta^{\omega\gamma}\eta^{\alpha\sigma}\partial_\alpha\partial_\omega h_{\sigma\gamma} + c_5\eta^{\omega\sigma}\partial_\mu\partial_\nu h_{\omega\sigma}\,.\end{aligned}$$

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- Terms involving c_1 and c_3 do not contribute for a null wave $\Box h_{\mu\nu} = 0$.

Newman-Penrose formalism

Field equations expressed in Newman-Penrose basis

$$
\begin{aligned}
0 &= E_{nn} = -2(c_2\ddot{h}_{ln} + c_4\ddot{h}_{nl} + c_5\ddot{h}_{nl} - c_5\ddot{h}_{m\bar{m}})\,,\\
0 &= E_{mn} = E_{nm} = -(c_2 + c_4)\ddot{h}_{lm}\,,\\
0 &= E_{\bar{m}n} = E_{n\bar{m}} = -(c_2 + c_4)\ddot{h}_{l\bar{m}}\,,\\
0 &= E_{m\bar{m}} = E_{\bar{m}m} = c_5\ddot{h}_{ll}\,,\\
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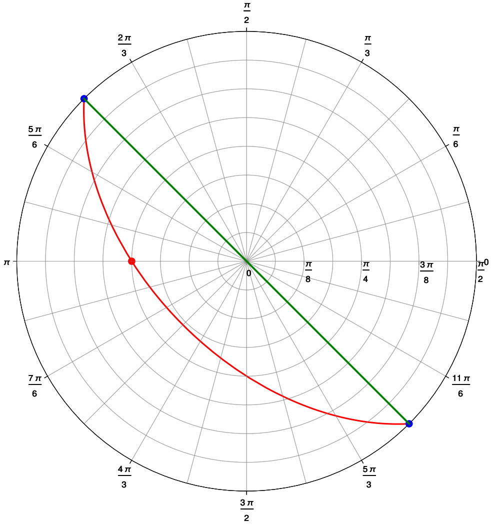

- N_2
- N_3
- III_5
- II_6

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- Symmetric teleparallel gravity:
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  - Most general theory needs 5 parameters at linearized level.
- Results:
  - Gravitational waves propagate at the speed of light (not shown in this talk).
  - Polarisation classes N_2, N_3, III_5, II_6: tensor modes always exist, maybe more.

Acknowledgments

Teleparallel gravity workshop

June 25-29, 2018 - Tartu, Estonia

http://hexagon.fi.tartu.ee/~telegrav2018/

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