PPN parameter γ for multiscalar-tensor gravity with a general potential [1607.02356]

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Post-Newtonian approximation

- Perfect fluid matter with density ρ, internal energy $\rho\Pi$, pressure p, velocity $v^i = \frac{u^i}{u^0}$:

$$T^{\mu\nu} = (\rho + \rho\Pi + p)u^\mu u^\nu + p g^{\mu\nu}\,.$$

- Relevant velocity orders $\mathcal{O}(n) \sim |\vec{v}|^n$:

$$g_{00} = -1 + \overset{(2)}{h}_{00} + \mathcal{O}(4)\,,$$
$$g_{ij} = \delta_{ij} + \overset{(2)}{h}_{ij} + \mathcal{O}(4)\,,$$
$$\Phi^\alpha = \overset{(0)}{\Phi}{}^\alpha + \overset{(2)}{\Phi}{}^\alpha + \mathcal{O}(4)\,.$$

- Gauge condition:

$$\overset{(2)}{h}{}_i{}^j{}_{,j} - \frac{1}{2}\overset{(2)}{h}{}_\mu{}^\mu{}_{,i} = \frac{1}{\mathcal{F}_0}\left.\frac{\partial\mathcal{F}}{\partial\Phi^\alpha}\right|_0 \overset{(2)}{\Phi}{}^\alpha{}_{,i}\,.$$

Point mass approximation

- Matter source:

$$\rho = M\delta(\vec{x})\,,\ \Pi = 0\,,\ p = 0\,,\ v_i = 0\,.$$

- Newtonian potential:

$$U(r) = \frac{M}{r}\,.$$

- Metric perturbation:

$$\overset{(2)}{h}_{00} = 2G_{\text{eff}}(r)U(r)\,,$$
$$\overset{(2)}{h}_{ij} = 2G_{\text{eff}}(r)\gamma(r)U(r)\delta_{ij}\,.$$

Multiscalar-tensor action and field equations

- Action for metric $g_{\mu\nu}$, N scalar fields Φ^α and matter fields χ_m:

$$S = \frac{1}{2\kappa^2}\int_{V_4} d^4x\sqrt{-g}\left(\mathcal{F}(\Phi)R - \mathcal{Z}_{\alpha\beta}(\Phi)g^{\mu\nu}\partial_\mu\Phi^\alpha\partial_\nu\Phi^\beta - 2\kappa^2\mathcal{U}(\Phi)\right) + S_m[g_{\mu\nu},\chi_m]\,.$$

- Metric field equation:

$$\mathcal{F}\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) + g_{\mu\nu}\Box\mathcal{F} - \nabla_\mu\nabla_\nu\mathcal{F} + \frac{1}{2}g_{\mu\nu}\mathcal{Z}_{\alpha\beta}\nabla_\rho\Phi^\alpha\nabla^\rho\Phi^\beta - \mathcal{Z}_{\alpha\beta}\nabla_\mu\Phi^\alpha\nabla_\nu\Phi^\beta + \kappa^2 g_{\mu\nu}\mathcal{U} = \kappa^2 T_{\mu\nu}\,.$$

- Scalar field equations:

$$\left(2\mathcal{F}\mathcal{Z}_{\alpha\beta} + 3\frac{\partial\mathcal{F}}{\partial\Phi^\alpha}\frac{\partial\mathcal{F}}{\partial\Phi^\beta}\right)\Box\Phi^\beta = -3\frac{\partial\mathcal{F}}{\partial\Phi^\alpha}\frac{\partial^2\mathcal{F}}{\partial\Phi^\beta\partial\Phi^\delta}\partial_\rho\Phi^\beta\partial^\rho\Phi^\delta - \frac{\partial\mathcal{F}}{\partial\Phi^\alpha}\mathcal{Z}_{\beta\delta}\partial_\rho\Phi^\beta\partial^\rho\Phi^\delta +$$
$$+ \mathcal{F}\frac{\partial\mathcal{Z}_{\beta\delta}}{\partial\Phi^\alpha}\partial_\rho\Phi^\beta\partial^\rho\Phi^\delta - 2\mathcal{F}\frac{\partial\mathcal{Z}_{\alpha\beta}}{\partial\Phi^\delta}\partial_\rho\Phi^\beta\partial^\rho\Phi^\delta - 4\frac{\partial\mathcal{F}}{\partial\Phi^\alpha}\kappa^2\mathcal{U} + 2\mathcal{F}\kappa^2\frac{\partial\mathcal{U}}{\partial\Phi^\alpha} + \frac{\partial\mathcal{F}}{\partial\Phi^\alpha}\kappa^2 T\,.$$

Scalar field at order $\mathcal{O}(2)$

- Structure of $\mathcal{O}(2)$ scalar equation:

$$\nabla^2\overset{(2)}{\Phi}{}^\alpha = \mathcal{M}^\alpha{}_\beta\overset{(2)}{\Phi}{}^\beta + k^\alpha\rho\,.$$

- Structure of solution per Jordan block:

$$\overset{(2)}{\Phi}{}^\alpha = -\frac{M}{4\pi r}\mathcal{E}^\alpha{}_\beta(r)k^\beta\,.$$

- Generalized matrix exponential:

$$\mathcal{E}(r) = \sum_{i=0}^{\infty}\left(\frac{\mathcal{M}^i r^{2i}}{(2i)!} - \frac{\sqrt{\mathcal{M}}^{2i+1}r^{2i+1}}{(2i+1)!}\right)\,,$$

where $\sqrt{\mathcal{M}} = 0$ if $\mathcal{M}$ has no square root.

Metric at order $\mathcal{O}(2)$

- Structure of $\mathcal{O}(2)$ metric field equation:

$$\nabla^2\left(\overset{(2)}{h}_{00} - \frac{1}{\mathcal{F}_0}\left.\frac{\partial\mathcal{F}}{\partial\Phi^\alpha}\right|_0\overset{(2)}{\Phi}{}^\alpha\right) = -\frac{\kappa^2}{\mathcal{F}_0}\rho\,,$$
$$\nabla^2\left(\overset{(2)}{h}_{ij} + \frac{1}{\mathcal{F}_0}\delta_{ij}\left.\frac{\partial\mathcal{F}}{\partial\Phi^\alpha}\right|_0\overset{(2)}{\Phi}{}^\alpha\right) = -\frac{\kappa^2}{\mathcal{F}_0}\delta_{ij}\rho\,.$$

- Read off observable parameters:

$$\boxed{G_{\text{eff}} = \frac{\kappa^2}{8\pi\mathcal{F}_0}(1-\Gamma(r))}\,,\quad \boxed{\gamma = \frac{1+\Gamma(r)}{1-\Gamma(r)}}\,,$$
$$\Gamma(r) = -\frac{4\mathcal{F}_0^2}{\kappa^4}k_\alpha\mathcal{E}^\alpha{}_\beta(r)k^\beta\,.$$

Example: two-field case in Brans-Dicke like parametrization

- Input parameters for $N = 2$ scalar fields $\Phi^1 = \phi, \Phi^2 = \Psi$:

$$\mathcal{F}(\Phi) = \Psi\,,\qquad \mathcal{Z}_{\alpha\beta}(\Phi) = \operatorname{diag}\left(Z(\phi,\Psi), \frac{\omega(\phi,\Psi)}{\Psi}\right)\,,\qquad \mathcal{U}(\Phi) = \mathcal{U}(\phi,\Psi)\,.$$

- Observable parameters:

$$\boxed{G_{\text{eff}} = \frac{\kappa^2}{8\pi\Psi_0}\left(1 + \frac{\cos^2\vartheta\, e^{-m_+r} + \sin^2\vartheta\, e^{-m_-r}}{2\omega_0+3}\right)}\,,\quad \boxed{\gamma = \frac{2\omega_0 + 3 - \cos^2\vartheta\, e^{-m_+r} - \sin^2\vartheta\, e^{-m_-r}}{2\omega_0 + 3 + \cos^2\vartheta\, e^{-m_+r} + \sin^2\vartheta\, e^{-m_-r}}}\,.$$

- Angle of non-minimal coupling and scalar field masses:

$$\cos^2\vartheta = \frac{1}{2}\left(1 + \frac{A}{B}\right)\,,\quad m_\pm^2 = \frac{\kappa^2}{2Z_0(2\omega_0+3)}\left((2\omega_0+3)\left.\frac{\partial^2\mathcal{U}}{\partial\phi^2}\right|_0 + 2\Psi_0 Z_0\left.\frac{\partial^2\mathcal{U}}{\partial\Psi^2}\right|_0 \pm B\right)\,,$$
$$A = 2\Psi_0 Z_0\left.\frac{\partial^2\mathcal{U}}{\partial\Psi^2}\right|_0 - (2\omega_0+3)\left.\frac{\partial^2\mathcal{U}}{\partial\phi^2}\right|_0\,,\quad B = \sqrt{A^2 + 8(2\omega_0+3)Z_0\Psi_0\left(\left.\frac{\partial^2\mathcal{U}}{\partial\phi\,\partial\Psi}\right|_0\right)^2}\,.$$

2σ bounds from Cassini tracking experiment on two-field Brans-Dicke

- Left to right:
  - $\vartheta = 0$,
  - $\vartheta = \pi/8$,
  - $\vartheta = \pi/4$.
- $\tilde{m}_\pm = m_\pm\sqrt{2\omega_0 + 3}$.
- Cassini bound:

  $\gamma - 1 = (2.1 \pm 2.3)\cdot 10^{-5}$

- Tightest bound on γ.
- Region left of surface is excluded at 2σ.