Parameterized post-Newtonian formalism for multimetric gravity

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Motivation

- ΛCDM model: 95% of the universe are dark matter / dark energy.
- Constituents of dark universe are unknown.
- Idea: DM / DE effects from additional *dark* standard model copies.
- Only interaction between standard model copies: repulsive gravity.
- Universe contains equal amounts of matter from all copies.

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- ΛCDM model: 95% of the universe are dark matter / dark energy.
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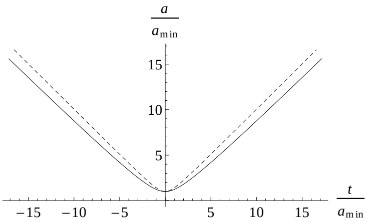
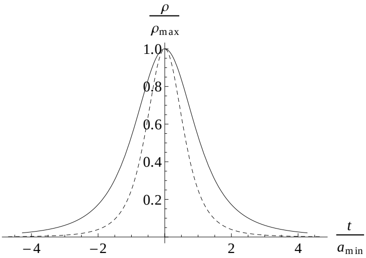
- Idea: DM / DE effects from additional *dark* standard model copies.
- Only interaction between standard model copies: repulsive gravity.
- Universe contains equal amounts of matter from all copies.
- Dark galaxies "push" visible matter & light towards visible galaxies.
  ⇒ Explanation of dark matter!
- Mutual repulsion between galaxies drives accelerating expansion.
  ⇒ Explanation of dark energy! [MH, M. Wohlfarth '10]





Construction principles of Multimetric gravity

- $N \geq 2$ standard model copies Ψ^I governed by metrics g^I.
- Each standard model copy Ψ^I couples only to its own metric g^I:

$$\Rightarrow \quad S_M[g^I, \Psi^I] = \int d^4x \sqrt{g^I} \mathcal{L}_M[g^I, \Psi^I] \,.$$

- Different sectors couple only gravitationally:

$$\Rightarrow \quad S = S_G[g^1, \ldots, g^N] + \sum_{I=1}^{N} S_M[g^I, \Psi^I] \,.$$

- Field equations obtained from variation with respect to g^I:

$$K^I_{ab} = 8\pi G_N T^I_{ab}$$

- Curvature tensor K^I_{ab} of second derivative order.
- Permutation symmetry of the sectors (g^I, Ψ^I).
- Vacuum solution given by flat metrics $g^I = \eta$.

Post-Newtonian bookkeeping

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- Matter in the solar system: perfect fluid with
  - density $\rho^I \sim \mathcal{O}(2)$
  - pressure $p^I \sim \mathcal{O}(4)$
  - specific internal energy $\Pi^I \sim \mathcal{O}(2)$
  - velocity $\vec{v}^I \sim \mathcal{O}(1)$

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  - velocity $\vec{v}^I \sim \mathcal{O}(1)$
- Weak gravitational field.

$\Rightarrow$ Expand metric around flat background:

$$g^I_{ab} = \eta_{ab} + h^I_{ab} = \eta_{ab} + h^{I(1)}_{ab} + h^{I(2)}_{ab} + h^{I(3)}_{ab} + h^{I(4)}_{ab}$$

- Each term $h^{I(n)}_{ab}$ is of order $\mathcal{O}(n)$.

Post-Newtonian metric

- Post-Newtonian metric ansatz:

$$h_{00}^{I(2)} = -\sum_{J=1}^{N} \alpha^{IJ} \triangle \chi^{J} \,,$$

$$h_{\alpha\beta}^{I(2)} = \sum_{J=1}^{N} \left(2\theta^{IJ} \chi^{J}{}_{,\alpha\beta} - (\gamma^{IJ} + \theta^{IJ}) \triangle \chi^{J} \delta_{\alpha\beta} \right) \,,$$

$$h_{0\alpha}^{I(3)} = \sum_{J=1}^{N} \left(\sigma_{+}^{IJ} W_{\alpha}^{J+} + \sigma_{-}^{IJ} W_{\alpha}^{J-} \right) \,,$$

$$h_{00}^{I(4)} = \sum_{J=1}^{N} \left(\phi_{p}^{IJ} \Phi_{p}^{J} + \phi_{\Pi}^{IJ} \Phi_{\Pi}^{J} + \sum_{A=1}^{2} \omega_{A}^{IJ} \Omega_{A}^{J} \right) + \sum_{J,K=1}^{N} \sum_{A=1}^{7} \psi_{A}^{IJK} \Psi_{A}^{JK} \,.$$

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- Parameters $\alpha^{IJ}, \gamma^{IJ}, \theta^{IJ}, \sigma_{\pm}^{IJ}, \phi_{p}^{IJ}, \phi_{\Pi}^{IJ}, \omega_{1}^{IJ}, \omega_{2}^{IJ}, \psi_{1}^{IJK}, \ldots, \psi_{7}^{IJK}$.

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- Parameters $\alpha^{IJ}, \gamma^{IJ}, \theta^{IJ}, \sigma_{\pm}^{IJ}, \phi_{p}^{IJ}, \phi_{\Pi}^{IJ}, \omega_{1}^{IJ}, \omega_{2}^{IJ}, \psi_{1}^{IJK}, \ldots, \psi_{7}^{IJK}$.
- Potentials $\chi^{I}, W^{I\pm}, \Phi_{p}^{I}, \Phi_{\Pi}^{I}, \Omega_{1}^{I}, \Omega_{2}^{I}, \Psi_{1}^{IJ}, \ldots, \Psi_{7}^{IJ}$.

- Superpotential:

$$\chi^I = -\int \rho'^I |\vec{x} - \vec{x}'| d^3x' \,.$$

- Vector potentials:

$$W_\alpha^{\pm I} = \int \rho'^I \left(\frac{v'^I_\alpha}{|\vec{x} - \vec{x}'|} \pm \frac{(x_\alpha - x'_\alpha)(x_\beta - x'_\beta) v'^I_\beta}{|\vec{x} - \vec{x}'|^3} \right) d^3x' \,.$$

- Pressure:

$$\Phi_p^I = \int \frac{p'^I}{|\vec{x} - \vec{x}'|} d^3x' \,.$$

- Internal energy:

$$\Phi_\Pi^I = \int \frac{\rho'^I \Pi'^I}{|\vec{x} - \vec{x}'|} d^3x' \,.$$

PPN potentials - part 2

- Kinetic energy:

$$\Omega_1^I = \int \frac{\rho'^I v'^{I2}}{|\vec{x} - \vec{x}'|} d^3x' \,, \quad \Omega_2^I = \int \frac{\rho'^I \left[\vec{v}'^I \cdot (\vec{x} - \vec{x}')\right]^2}{|\vec{x} - \vec{x}'|^3} d^3x' \,.$$

- Non-linear potentials:

$$\triangle\triangle\Psi_1^{IJ} = \triangle\chi^I \triangle\triangle\triangle\chi^J \,, \qquad \triangle\triangle\Psi_2^{IJ} = \chi^I_{,\alpha\beta}\triangle\triangle\chi^J_{,\alpha\beta} \,,$$
$$\triangle\triangle\Psi_3^{IJ} = \triangle\chi^I_{,\alpha}\triangle\triangle\chi^J_{,\alpha} \,, \qquad \triangle\triangle\Psi_4^{IJ} = \chi^I_{,\alpha\beta\gamma}\triangle\chi^J_{,\alpha\beta\gamma} \,,$$
$$\triangle\triangle\Psi_5^{IJ} = \triangle\triangle\chi^I \triangle\triangle\chi^J \,, \qquad \triangle\triangle\Psi_6^{IJ} = \triangle\chi^I_{,\alpha\beta}\triangle\chi^J_{,\alpha\beta} \,,$$
$$\triangle\triangle\Psi_7^{IJ} = \chi^I_{,\alpha\beta\gamma\delta}\chi^J_{,\alpha\beta\gamma\delta} \,.$$

Relation to standard PPN parameters

- PPN parameters relevant for visible matter T^1_{ab} and metric g^1_{ab}:

$$\alpha^{11} = \alpha\,, \quad \sigma_-^{11} = -\frac{1}{2} - \gamma - \frac{1}{4}\alpha_1 + \frac{1}{2}\alpha_2 - \frac{1}{2}\zeta_1 + \xi\,,$$
$$\gamma^{11} = \gamma\,, \quad \sigma_+^{11} = -1 - \gamma - \frac{1}{4}\alpha_1\,, \quad \phi_\Pi^{11} = 2 + 2\zeta_3\,,$$
$$\phi_p^{11} = 6\gamma + 6\zeta_4 + 4\xi\,, \quad \omega_1^{11} = 2 + 2\gamma + \alpha_3 + \zeta_1 - 2\xi\,,$$
$$\omega_2^{11} = 2\xi - \zeta_1\,, \quad \psi_2^{111} = 2\xi\,, \quad \psi_6^{111} = 6\xi - 2\beta\,,$$
$$\psi_1^{111} = \psi_5^{111} = \frac{1}{2} + \frac{3}{2}\gamma - 2\beta + \frac{1}{2}\zeta_2 + \xi\,, \quad \psi_4^{111} = 4\xi\,,$$
$$\psi_3^{111} = 1 + 3\gamma - 6\beta + \zeta_2 + 6\xi\,, \quad \theta^{11} = \psi_7^{111} = 0\,.$$

Relation to standard PPN parameters

- PPN parameters relevant for visible matter T^1_{ab} and metric g^1_{ab}:

$$\alpha^{11} = \alpha = 1\,, \quad \sigma^{11}_{-} = -\frac{1}{2} - \gamma - \frac{1}{4}\alpha_1 + \frac{1}{2}\alpha_2 - \frac{1}{2}\zeta_1 + \xi = -\frac{3}{2}\,,$$

$$\gamma^{11} = \gamma = 1\,, \quad \sigma^{11}_{+} = -1 - \gamma - \frac{1}{4}\alpha_1 = -2\,, \quad \phi^{11}_{\Pi} = 2 + 2\zeta_3 = 2\,,$$

$$\phi^{11}_{p} = 6\gamma + 6\zeta_4 + 4\xi = 6\,, \quad \omega^{11}_{1} = 2 + 2\gamma + \alpha_3 + \zeta_1 - 2\xi = 4\,,$$

$$\omega^{11}_{2} = 2\xi - \zeta_1 = 0\,, \quad \psi^{111}_{2} = 2\xi = 0\,, \quad \psi^{111}_{6} = 6\xi - 2\beta = -2\,,$$

$$\psi^{111}_{1} = \psi^{111}_{5} = \frac{1}{2} + \frac{3}{2}\gamma - 2\beta + \frac{1}{2}\zeta_2 + \xi = 0\,, \quad \psi^{111}_{4} = 4\xi = 0\,,$$

$$\psi^{111}_{3} = 1 + 3\gamma - 6\beta + \zeta_2 + 6\xi = -2\,, \quad \theta^{11} = \psi^{111}_{7} = 0\,.$$

- Compare with measured values of standard PPN parameters.

⇒ $\theta^{11} = 0$ and $\psi^{111}_1 = \psi^{111}_5$ due to gauge freedom.

⇒ $\alpha^{11} = 1$ can be achieved by choice of units.

⇒ 13 physical parameters accessible to visible matter experiments.

Example action - part 1

Generic, simple, multimetric gravity action:

$$S_G = \frac{1}{16\pi} \int d^4x$$

$$\sqrt{g_0} \Bigg[\sum_{I=1}^{N} \Big(c_1 R^I + g^{I\,ij} \Big(c_3 \tilde{S}^I{}_i \tilde{S}^I{}_j + c_5 \tilde{S}^I{}_k \tilde{S}^{I\,k}{}_{ij} + c_7 \tilde{S}^{I\,k}{}_{il} \tilde{S}^{I\,l}{}_{jk} \Big)$$

$$+ g^{I\,ij} g^{I\,kl} g^I{}_{mn} \Big(c_9 \tilde{S}^{I\,m}{}_{ik} \tilde{S}^{I\,n}{}_{jl} + c_{11} \tilde{S}^{I\,m}{}_{ij} \tilde{S}^{I\,n}{}_{kl} \Big) \Big)$$

$$+ \sum_{I,J=1}^{N} \Big(c_2 g^{I\,ij} R^J{}_{ij} + g^{I\,ij} \Big(c_4 S^{IJ}{}_i S^{IJ}{}_j + c_6 S^{IJ}{}_k S^{IJ\,k}{}_{ij} + c_8 S^{IJ\,k}{}_{il} S^{IJ\,l}{}_{jk} \Big)$$

$$+ g^{I\,ij} g^{I\,kl} g^I{}_{mn} \Big(c_{10} S^{IJ\,m}{}_{ik} S^{IJ\,n}{}_{jl} + c_{12} S^{IJ\,m}{}_{ij} S^{IJ\,n}{}_{kl} \Big) \Big) \Bigg] .$$

Example action - part 2

- Ricci tensor and Ricci scalar:

$$R^I\,, \qquad g^{I\,ab}R^J{}_{ab}\,.$$

- Connection difference tensors:

$$S^{IJ\,i}{}_{jk} = \Gamma^{I\,i}{}_{jk} - \Gamma^{J\,i}{}_{jk}\,, \qquad S^{IJ}{}_j = S^{IJ\,k}{}_{jk}\,,$$

$$\tilde{S}^{J\,i}{}_{jk} = \frac{1}{N}\sum_{I=1}^{N} S^{IJ\,i}{}_{jk}\,, \qquad \tilde{S}^J{}_j = \tilde{S}^{J\,k}{}_{jk}\,.$$

- Mixed volume form

$$g_0 = \prod_{I=1}^{N}\left(g^I\right)^{\frac{1}{N}}\,.$$

- 12 free, constant parameters:

$$c_1, \ldots, c_{12}\,.$$

Repulsive Newtonian limit

- All diagonal elements α^{II} are equal.
- Units can be chosen to rescale $\alpha^{II} = 1$.
- All off-diagonal elements $\alpha^{IJ} = z$ are equal.
- Parameter values:

$$\alpha^{IJ} = \frac{\bar{\alpha}}{N} + \hat{\alpha}\delta^{IJ}$$

with

$$\bar{\alpha} = Nz\,, \qquad \hat{\alpha} = 1 - z\,.$$

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- Newtonian metric perturbation:

$$h_{00}^{I(2)} = -\triangle\chi^I - z\sum_{J\neq I}\triangle\chi^J$$
$$= 2U^I + 2z\sum_{J\neq I}U^J\,.$$

⇒ Repulsive Newtonian limit for $z = -1$.

Results

- 6 constants $c_4, c_6, c_8, c_{10}, c_{11}, c_{12}$ remain free parameters.
- 6 constants $c_1, c_2, c_3, c_5, c_7, c_9$ fixed depending on N and z.
- Multimetric gravity compatible with experiments.

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- Multimetric gravity compatible with experiments.
- PPN parameters (+ lengthy expressions for $\psi_1^{IJK}, \ldots, \psi_7^{IJK}$):

$$
\begin{aligned}
\bar{\alpha} &= Nz\,, & \hat{\alpha} &= 1 - z\,,\\
\bar{\gamma} &= Nz\,, & \hat{\gamma} &= 1 - z\,,\\
\bar{\theta} &= 0\,, & \hat{\theta} &= 0\,,\\
\bar{\sigma}_+ &= -2Nz\,, & \hat{\sigma}_+ &= 2(z-1)\,,\\
\bar{\sigma}_- &= \frac{N}{2}(1-4z)\,, & \hat{\sigma}_- &= 2(z-1)\,,\\
\bar{\omega}_1 &= N(5z-1)\,, & \hat{\omega}_1 &= 5(1-z)\,,\\
\bar{\omega}_2 &= N(1-z)\,, & \hat{\omega}_2 &= z-1\,,\\
\bar{\phi}_p &= 2N(4z-1)\,, & \hat{\phi}_p &= 8(1-z)\,,\\
\bar{\phi}_\Pi &= 2Nz\,, & \hat{\phi}_\Pi &= 2(1-z)\,.
\end{aligned}
$$

Summary

- Multimetric PPN formalism:
  - Assume perfect fluid matter.
  - Determine post-Newtonian metric.
  - PPN parameters $\alpha^{IJ}, \gamma^{IJ}, \theta^{IJ}, \sigma_{\pm}^{IJ}, \phi_{p}^{IJ}, \phi_{\Pi}^{IJ}, \omega_{1}^{IJ}, \omega_{2}^{IJ}, \psi_{1}^{IJK}, \ldots, \psi_{7}^{IJK}$.
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  - 13 parameters accessible to visible matter experiments.
  - Extension of standard PPN formalism to $N \geq 2$ metrics.
- Application to multimetric repulsive gravity:
  - Simple model dependent on 12 constant parameters.
  - 6 parameters fixed by experimental consistency.
  - Experimentally consistent model with 6 free parameters.
  - Repulsive gravity allows for accelerating expansion.

Outlook

- Experimental significance of new visible PPN parameters:
  - Effects of new PPN potentials on current experiments?
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  - New experiments to determine unknown parameters?
- Post-Newtonian conservation laws:
  - Conservation of total energy and momentum?
  - Transfer of energy and momentum between matter sectors?
- Extend other formalisms to multimetric gravity:
  - Strong fields and pulsars: parameterized post-Keplerian formalism?
  - Gravitational waves: parameterized post-Einsteinian formalism?