Experimental consistency of multimetric gravity

An extension of the PPN formalism to $N \geq 2$ metrics

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Outline

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- ΛCDM model: only 5% visible matter.
  - Dark matter explains “missing mass” in galaxies.
  - Dark energy explains accelerating expansion.
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- Idea here: Additional “dark, negative mass” standard model copy.
- Only interaction between both copies: repulsive gravity.
- Universe contains equal amounts of both types of mass:

$\rightsquigarrow$ Dark galaxies “push” visible matter & light towards visible galaxies.
$\Rightarrow$ Explanation of dark matter!

$\rightsquigarrow$ Mutual repulsion between galaxies drives accelerating expansion.
$\Rightarrow$ Explanation of dark energy!

Multimetric gravity

- No-go theorem for bimetric repulsive gravity. [MH, M. Wohlfarth '09]
- Solution: multimetric gravity with...
  - $N > 2$ metric tensors g^I_{ab}
  - $N > 2$ copies of standard model matter φ^I

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- Experimental consistency at full post-Newtonian level?

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- Slow-moving source matter.

$\Rightarrow$ Expand quantities in “velocity” orders $\mathcal{O}(n)$.

- Weak gravitational field.

$\Rightarrow$ Expand metric around flat background:

$$g^I_{ab} = \eta_{ab} + h^I_{ab} = \eta_{ab} + h^{I(1)}_{ab} + h^{I(2)}_{ab} + h^{I(3)}_{ab} + h^{I(4)}_{ab}$$

- Each term $h^{I(n)}_{ab}$ is of order $\mathcal{O}(n)$.

Post-Newtonian metric

- Post-Newtonian metric ansatz:

$$h^{I(2)}_{00} = -\sum_{J=1}^{N} \alpha^{IJ} \triangle \chi^J \,,$$

$$h^{I(2)}_{\alpha\beta} = \sum_{J=1}^{N} \left(2\theta^{IJ} \chi^J{}_{,\alpha\beta} - (\gamma^{IJ} + \theta^{IJ}) \triangle \chi^J \delta_{\alpha\beta} \right) \,,$$

$$h^{I(3)}_{0\alpha} = \sum_{J=1}^{N} \left(\sigma_+^{IJ} W_\alpha^{J+} + \sigma_-^{IJ} W_\alpha^{J-} \right) \,,$$

$$h^{I(4)}_{00} = \sum_{J=1}^{N} \left(\phi_p^{IJ} \Phi_p^J + \phi_\Pi^{IJ} \Phi_\Pi^J + \sum_{A=1}^{2} \omega_A^{IJ} \Omega_A^J \right) + \sum_{J,K=1}^{N} \sum_{A=1}^{7} \psi_A^{IJK} \Psi_A^{JK} \,.$$

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- Parameters $\alpha^{IJ}, \gamma^{IJ}, \theta^{IJ}, \sigma_\pm^{IJ}, \phi_p^{IJ}, \phi_\Pi^{IJ}, \omega_1^{IJ}, \omega_2^{IJ}, \psi_1^{IJK}, \ldots, \psi_7^{IJK}$.

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$$h_{0\alpha}^{I(3)} = \sum_{J=1}^{N} \left(\sigma_{+}^{IJ} W_{\alpha}^{J+} + \sigma_{-}^{IJ} W_{\alpha}^{J-}\right) \,,$$

$$h_{00}^{I(4)} = \sum_{J=1}^{N} \left(\phi_{p}^{IJ} \Phi_{p}^{J} + \phi_{\Pi}^{IJ} \Phi_{\Pi}^{J} + \sum_{A=1}^{2} \omega_{A}^{IJ} \Omega_{A}^{J}\right) + \sum_{J,K=1}^{N} \sum_{A=1}^{7} \psi_{A}^{IJK} \Psi_{A}^{JK} \,.$$

- Parameters $\alpha^{IJ}, \gamma^{IJ}, \theta^{IJ}, \sigma_{\pm}^{IJ}, \phi_{p}^{IJ}, \phi_{\Pi}^{IJ}, \omega_{1}^{IJ}, \omega_{2}^{IJ}, \psi_{1}^{IJK}, \ldots, \psi_{7}^{IJK}$.
- Potentials $\chi^{I}, W^{I\pm}, \Phi_{p}^{I}, \Phi_{\Pi}^{I}, \Omega_{1}^{I}, \Omega_{2}^{I}, \Psi_{1}^{IJ}, \ldots, \Psi_{7}^{IJ}$.

PPN potentials - part 1

- Superpotential:

$$\chi^I = -\int \rho'^I |\vec{x} - \vec{x}'| d^3x' \,.$$

- Vector potentials:

$$W_\alpha^{\pm I} = \int \rho'^I \left(\frac{v_\alpha'^I}{|\vec{x} - \vec{x}'|} \pm \frac{(x_\alpha - x'_\alpha)(x_\beta - x'_\beta) v_\beta'^I}{|\vec{x} - \vec{x}'|^3} \right) d^3x' \,.$$

- Pressure:

$$\Phi_p^I = \int \frac{p'^I}{|\vec{x} - \vec{x}'|} d^3x' \,.$$

- Internal energy:

$$\Phi_\Pi^I = \int \frac{\rho'^I \Pi'^I}{|\vec{x} - \vec{x}'|} d^3x' \,.$$

PPN potentials - part 2

- Kinetic energy:

$$\Omega_1^I = \int \frac{\rho'^I v'^{I2}}{|\vec{x} - \vec{x}'|} d^3x' \,, \quad \Omega_2^I = \int \frac{\rho'^I \left[\vec{v}'^I \cdot (\vec{x} - \vec{x}')\right]^2}{|\vec{x} - \vec{x}'|^3} d^3x' \,.$$

- Non-linear potentials:

$$\triangle\triangle\Psi_1^{IJ} = \triangle\chi^I\triangle\triangle\triangle\chi^J \,, \qquad \triangle\triangle\Psi_2^{IJ} = \chi^I_{,\alpha\beta}\triangle\triangle\chi^J_{,\alpha\beta} \,,$$
$$\triangle\triangle\Psi_3^{IJ} = \triangle\chi^I_{,\alpha}\triangle\triangle\chi^J_{,\alpha} \,, \qquad \triangle\triangle\Psi_4^{IJ} = \chi^I_{,\alpha\beta\gamma}\triangle\chi^J_{,\alpha\beta\gamma} \,,$$
$$\triangle\triangle\Psi_5^{IJ} = \triangle\triangle\chi^I\triangle\triangle\chi^J \,, \qquad \triangle\triangle\Psi_6^{IJ} = \triangle\chi^I_{,\alpha\beta}\triangle\chi^J_{,\alpha\beta} \,,$$
$$\triangle\triangle\Psi_7^{IJ} = \chi^I_{,\alpha\beta\gamma\delta}\chi^J_{,\alpha\beta\gamma\delta} \,.$$

Matter content

- Perfect fluid energy-momentum tensor:

$$T^I_{00} = \rho^I \left(1 + \Pi^I + {v^I}^2 + \sum_{J=1}^{N} \alpha^{IJ} \triangle\chi^J\right) + \mathcal{O}(6)\,,$$
$$T^I_{0\alpha} = -\rho^I v^I_\alpha + \mathcal{O}(5)\,,$$
$$T^I_{\alpha\beta} = \rho^I v^I_\alpha v^I_\beta + p^I \delta_{\alpha\beta} + \mathcal{O}(6)\,.$$

- Conservation laws:

$$0 = \nabla^I_a T^{I\,a0} = \rho^I_{,0} + (\rho^I v^I_\alpha)_{,\alpha} + \mathcal{O}(5)\,,$$
$$0 = \nabla^I_a T^{I\,a\alpha} = \rho^I \frac{dv^I_\alpha}{dt} + p^I_{,\alpha} + \frac{1}{2}\rho^I \sum_{J=1}^{N} \alpha^{IJ} \triangle\chi^J_{,\alpha} + \mathcal{O}(6)\,.$$

Gauge transformations

- Invariance of the action under diffeomorphisms.
- Diffeomorphism generated by vector field ξ.
- Tensor fields change by their Lie derivatives:

$$\delta_\xi g^I_{ab} = (\mathcal{L}_\xi g^I)_{ab} = 2\nabla^I_{(a}\xi_{b)}\,.$$

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- Require form-invariance of the PPN metric.

$\Rightarrow$ Vector field must take the form

$$\xi_0 = \sum_{I=1}^{N} \lambda^I_1 \chi^I_{,0}\,, \quad \xi_\alpha = \sum_{I=1}^{N} \lambda^I_2 \chi^I_{,\alpha}\,.$$

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- Use gauge invariance to eliminate potentials from the PPN metric.

⇒ PPN parameters $\theta^{II} = 0$ and $\psi^{III}_1 = \psi^{III}_5$ in standard gauge.

Lorentz transformations

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- Express PPN potentials in new coordinate system.

$\Rightarrow$ New $\vec{w}$ dependent terms in the PPN metric appear.

$\Rightarrow$ New terms vanish if and only if

$$\begin{aligned}
\alpha^{IJ} + \gamma^{IJ} + \theta^{IJ} + \sigma_+^{IJ} &= 0\,,\\
2\sigma_+^{IJ} + \omega_1^{IJ} + \omega_2^{IJ} &= 0\,,\\
\alpha^{IJ} + 2\theta^{IJ} - 2\sigma_-^{IJ} - \omega_1^{IJ} &= 0\,,\\
2\theta^{IJ} + \sigma_+^{IJ} - \sigma_-^{IJ} - 2\theta^{II} - \sigma_+^{II} + \sigma_-^{II} &= 0\,.
\end{aligned}$$

$\Rightarrow$ Simple test for Lorentz invariance of a gravity theory.

Order-wise solution of field equations

- Second velocity order $\mathcal{O}(2)$:
  - Solve field equations $K_{00}^{I(2)} = 8\pi T_{00}^{I(2)}$ and $K_{\alpha\beta}^{I(2)} = 8\pi T_{\alpha\beta}^{I(2)}$.
  - Determine metric components $h_{00}^{I(2)}$ and $h_{\alpha\beta}^{I(2)}$.

  $\Rightarrow$ Obtain PPN parameters $\alpha^{IJ}, \gamma^{IJ}, \theta^{IJ}$.

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  - $\Rightarrow$ Obtain PPN parameters $\alpha^{IJ}, \gamma^{IJ}, \theta^{IJ}$.
- Third velocity order $\mathcal{O}(3)$:
  - Solve field equations $K^{I(3)}_{0\alpha} = 8\pi T^{I(3)}_{0\alpha}$.
  - Determine metric components $h^{I(3)}_{0\alpha}$.
  - $\Rightarrow$ Obtain PPN parameters σ^{IJ}_{+}.

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  - Determine metric components $h^{I(3)}_{0\alpha}$.

  $\Rightarrow$ Obtain PPN parameters σ^{IJ}_{+}.
- Fourth velocity order $\mathcal{O}(4)$:
  - Solve field equations $K^{I(4)}_{00} = 8\pi T^{I(4)}_{00}$ and $K^{I(4)}_{\alpha\beta} = 8\pi T^{I(4)}_{\alpha\beta}$.
  - Determine metric component $h^{I(4)}_{00}$.

  $\Rightarrow$ Obtain PPN parameters $\sigma^{IJ}_{-}, \phi^{IJ}_{p}, \phi^{IJ}_{\Pi}, \omega^{IJ}_{1}, \omega^{IJ}_{2}, \psi^{IJK}_{1}, \ldots, \psi^{IJK}_{7}$.

Outline

Standard PPN formalism

- Only one metric g_{ab} and corresponding matter source T_{ab}.
- Standard PPN metric:

$$\begin{aligned} h^{(2)}_{00} &= 2\alpha U\,, \\ h^{(2)}_{\alpha\beta} &= 2\gamma U\delta_{\alpha\beta}\,, \\ h^{(3)}_{0\alpha} &= -\frac{1}{2}(3+4\gamma+\alpha_1-\alpha_2+\zeta_1-2\xi)V_\alpha \\ &\quad -\frac{1}{2}(1+\alpha_2-\zeta_1+2\xi)W_\alpha\,, \\ h^{(4)}_{00} &= -2\beta U^2-2\xi\Phi_W+(2+2\gamma+\alpha_3+\zeta_1-2\xi)\Phi_1 \\ &\quad +2(1+3\gamma-2\beta+\zeta_2+\xi)\Phi_2+2(1+\zeta_3)\Phi_3 \\ &\quad +2(3\gamma+3\zeta_4-2\xi)\Phi_4-(\zeta_1-2\xi)\mathcal{A}\,. \end{aligned}$$

- Identify $g_{ab} \equiv g^1_{ab}$ and $T_{ab} = T^1_{ab}$.

Translation of PPN potentials

- Standard in terms of multimetric PPN potentials:

$$U = -\frac{1}{2}\triangle\chi^1\,, \quad U^2 = \frac{1}{2}\Psi_1^{11} + 2\Psi_3^{11} + \frac{1}{2}\Psi_5^{11} + \Psi_6^{11}\,,$$

$$V_\alpha = \frac{W_\alpha^{+1} + W_\alpha^{-1}}{2}\,, \quad W_\alpha = \frac{W_\alpha^{+1} - W_\alpha^{-1}}{2}\,,$$

$$\Phi_1 = \Omega_1^1\,, \quad \Phi_2 = \frac{1}{4}\Psi_1^{11} + \frac{1}{2}\Psi_3^{11} + \frac{1}{4}\Psi_5^{11}\,, \quad \Phi_3 = \Phi_\Pi^1\,, \quad \Phi_4 = \Phi_p^1\,,$$

$$\Phi_W = -\frac{1}{4}\Psi_1^{11} - \Psi_2^{11} - \frac{5}{2}\Psi_3^{11} - 2\Psi_4^{11} - \frac{1}{4}\Psi_5^{11} - 3\Psi_6^{11}\,, \quad \mathcal{A} = \Omega_2^1\,.$$

- Larger number of multimetric vs. standard potentials.

⇒ Cannot express all multimetric in terms of standard potentials.

⇒ Multimetric PPN formalism is more general.

Translation of PPN parameters

- Multimetric in terms of standard PPN parameters:

$$\alpha^{11} = \alpha\,, \quad \sigma_-^{11} = -\frac{1}{2} - \gamma - \frac{1}{4}\alpha_1 + \frac{1}{2}\alpha_2 - \frac{1}{2}\zeta_1 + \xi\,,$$
$$\gamma^{11} = \gamma\,, \quad \sigma_+^{11} = -1 - \gamma - \frac{1}{4}\alpha_1\,, \quad \phi_\Pi^{11} = 2 + 2\zeta_3\,,$$
$$\phi_p^{11} = 6\gamma + 6\zeta_4 + 4\xi\,, \quad \omega_1^{11} = 2 + 2\gamma + \alpha_3 + \zeta_1 - 2\xi\,,$$
$$\omega_2^{11} = 2\xi - \zeta_1\,, \quad \psi_2^{111} = 2\xi\,, \quad \psi_6^{111} = 6\xi - 2\beta\,,$$
$$\psi_1^{111} = \psi_5^{111} = \frac{1}{2} + \frac{3}{2}\gamma - 2\beta + \frac{1}{2}\zeta_2 + \xi\,, \quad \psi_4^{111} = 4\xi\,,$$
$$\psi_3^{111} = 1 + 3\gamma - 6\beta + \zeta_2 + 6\xi\,, \quad \theta^{11} = \psi_7^{111} = 0\,.$$

Translation of PPN parameters

- Multimetric in terms of standard PPN parameters:

$$\alpha^{11} = \alpha = 1\,, \quad \sigma_-^{11} = -\frac{1}{2} - \gamma - \frac{1}{4}\alpha_1 + \frac{1}{2}\alpha_2 - \frac{1}{2}\zeta_1 + \xi = -\frac{3}{2}\,,$$

$$\gamma^{11} = \gamma = 1\,, \quad \sigma_+^{11} = -1 - \gamma - \frac{1}{4}\alpha_1 = -2\,, \quad \phi_\Pi^{11} = 2 + 2\zeta_3 = 2\,,$$

$$\phi_p^{11} = 6\gamma + 6\zeta_4 + 4\xi = 6\,, \quad \omega_1^{11} = 2 + 2\gamma + \alpha_3 + \zeta_1 - 2\xi = 4\,,$$

$$\omega_2^{11} = 2\xi - \zeta_1 = 0\,, \quad \psi_2^{111} = 2\xi = 0\,, \quad \psi_6^{111} = 6\xi - 2\beta = -2\,,$$

$$\psi_1^{111} = \psi_5^{111} = \frac{1}{2} + \frac{3}{2}\gamma - 2\beta + \frac{1}{2}\zeta_2 + \xi = 0\,, \quad \psi_4^{111} = 4\xi = 0\,,$$

$$\psi_3^{111} = 1 + 3\gamma - 6\beta + \zeta_2 + 6\xi = -2\,, \quad \theta^{11} = \psi_7^{111} = 0\,.$$

- Compare with measured values of standard PPN parameters.

⇒ $\theta^{11} = 0$ and $\psi_1^{111} = \psi_5^{111}$ due to gauge choice.

⇒ $\alpha^{11} = 1$ can be achieved by choice of units.

⇒ 13 physical parameters accessible to visible matter experiments.

Outline

Action - part 1

Generic, simple, multimetric gravity action:

$$
\begin{aligned}
S_G = \frac{1}{16\pi}\int d^4x \\
\sqrt{g_0}\Bigg[\sum_{I=1}^{N}\bigg(c_1 R^I + g^{I\,ij}\Big(c_3 \tilde{S}^I{}_i \tilde{S}^I{}_j + c_5 \tilde{S}^I{}_k \tilde{S}^{I\,k}{}_{ij} + c_7 \tilde{S}^{I\,k}{}_{il} \tilde{S}^{I\,l}{}_{jk}\Big) \\
+ g^{I\,ij} g^{I\,kl} g^I{}_{mn}\Big(c_9 \tilde{S}^{I\,m}{}_{ik} \tilde{S}^{I\,n}{}_{jl} + c_{11} \tilde{S}^{I\,m}{}_{ij} \tilde{S}^{I\,n}{}_{kl}\Big)\bigg) \\
+ \sum_{I,J=1}^{N}\bigg(c_2 g^{I\,ij} R^J{}_{ij} + g^{I\,ij}\Big(c_4 S^{IJ}{}_i S^{IJ}{}_j + c_6 S^{IJ}{}_k S^{IJ\,k}{}_{ij} + c_8 S^{IJ\,k}{}_{il} S^{IJ\,l}{}_{jk}\Big) \\
+ g^{I\,ij} g^{I\,kl} g^I{}_{mn}\Big(c_{10} S^{IJ\,m}{}_{ik} S^{IJ\,n}{}_{jl} + c_{12} S^{IJ\,m}{}_{ij} S^{IJ\,n}{}_{kl}\Big)\bigg)\Bigg].
\end{aligned}
$$

Action - part 2

- Ricci tensor and Ricci scalar:

$$R^I \,, \qquad g^{I\,ab} R^J{}_{ab} \,.$$

- Connection difference tensors:

$$S^{IJ\,i}{}_{jk} = \Gamma^{I\,i}{}_{jk} - \Gamma^{J\,i}{}_{jk} \,, \qquad S^{IJ}{}_j = S^{IJ\,k}{}_{jk} \,,$$

$$\tilde{S}^{J\,i}{}_{jk} = \frac{1}{N} \sum_{I=1}^{N} S^{IJ\,i}{}_{jk} \,, \qquad \tilde{S}^J{}_j = \tilde{S}^{J\,k}{}_{jk} \,.$$

- Mixed volume form

$$g_0 = \prod_{I=1}^{N} \left(g^I\right)^{\frac{1}{N}} \,.$$

- 12 free, constant parameters:

$$c_1, \ldots, c_{12} \,.$$

Permutation symmetry

- Consider permutation $I \mapsto \pi(I) = \tilde{I}$ of sectors.
- Action is symmetric under arbitrary permutations:

$$S[g^I, \varphi^I] = S[g^{\tilde{I}}, \varphi^{\tilde{I}}]\,.$$

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- All constant expansion coefficients $P^{I_1 \cdots I_n}$ are symmetric:

$$P^{I_1 \cdots I_n} = P^{\tilde{I}_1 \cdots \tilde{I}_n}\,.$$

- Most general form for 2 or 3 indices:

$$P_2^{IJ} = \frac{\bar{P}_2}{N} + \hat{P}_2 \delta^{IJ}\,,$$

$$P_3^{IJK} = \frac{\bar{P}_3}{N^2} + \frac{\overleftarrow{P}_3 \delta^{IJ} + \vec{P}_3 \delta^{IK} + \tilde{P}_3 \delta^{JK}}{N} + \hat{P}_3 \delta^{IJ} \delta^{IK}\,.$$

Repulsive Newtonian limit

- All diagonal elements α^{II} are equal.
- Units can be chosen to rescale $\alpha^{II} = 1$.
- All off-diagonal elements $\alpha^{IJ} = z$ are equal.
- Parameter values:

$$\bar{\alpha} = Nz \,, \qquad \hat{\alpha} = 1 - z \,.$$

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- All off-diagonal elements $\alpha^{IJ} = z$ are equal.
- Parameter values:

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- Newtonian metric perturbation:

$$\begin{aligned} h^{I(2)}_{00} &= -\triangle\chi^I - z\sum_{J\neq I}\triangle\chi^J \\ &= 2U^I + 2z\sum_{J\neq I}U^J \,. \end{aligned}$$

$\Rightarrow$ Repulsive Newtonian limit for $z = -1$.

Conditions

- Gauge fixing conditions for standard PPN gauge:

$$\frac{\bar{\theta}}{N} + \hat{\theta} = 0\,,$$

$$\frac{\bar{\psi}_1}{N^2} + \frac{\overleftarrow{\psi}_1 + \vec{\psi}_1 + \tilde{\psi}_1}{N} + \hat{\psi}_1 = \frac{\bar{\psi}_5}{N^2} + \frac{\overleftarrow{\psi}_5 + \vec{\psi}_5 + \tilde{\psi}_5}{N} + \hat{\psi}_5\,.$$

⇒ Provide independent equations for calculating PPN parameters.

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$\Rightarrow$ Provide independent equations for calculating PPN parameters.

- Experimental consistency with solar system:

$$\frac{\bar{\gamma}}{N} + \hat{\gamma} = 1\,,$$

$$\frac{\bar{\psi}_1}{N^2} + \frac{\overleftarrow{\psi}_1 + \overrightarrow{\psi}_1 + \tilde{\psi}_1}{N} + \hat{\psi}_1 = 0\,,$$

+ similar constraints from other PPN parameters.

$\Rightarrow$ Impose restrictions on viable input parameters $c_1, \ldots, c_{12}$.

Results

- 6 constants $c_4, c_6, c_8, c_{10}, c_{11}, c_{12}$ remain free parameters.
- 6 constants $c_1, c_2, c_3, c_5, c_7, c_9$ fixed depending on N and z.
- Multimetric gravity compatible with experiments.

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- 6 constants $c_1, c_2, c_3, c_5, c_7, c_9$ fixed depending on N and z.
- Multimetric gravity compatible with experiments.
- PPN parameters (+ lengthy expressions for $\psi_1^{IJK}, \ldots, \psi_7^{IJK}$):

$$
\begin{aligned}
\bar{\alpha} &= Nz\,, & \hat{\alpha} &= 1 - z\,,\\
\bar{\gamma} &= Nz\,, & \hat{\gamma} &= 1 - z\,,\\
\bar{\theta} &= 0\,, & \hat{\theta} &= 0\,,\\
\bar{\sigma}_+ &= -2Nz\,, & \hat{\sigma}_+ &= 2(z-1)\,,\\
\bar{\sigma}_- &= \frac{N}{2}(1-4z)\,, & \hat{\sigma}_- &= 2(z-1)\,,\\
\bar{\omega}_1 &= N(5z-1)\,, & \hat{\omega}_1 &= 5(1-z)\,,\\
\bar{\omega}_2 &= N(1-z)\,, & \hat{\omega}_2 &= z-1\,,\\
\bar{\phi}_p &= 2N(4z-1)\,, & \hat{\phi}_p &= 8(1-z)\,,\\
\bar{\phi}_\Pi &= 2Nz\,, & \hat{\phi}_\Pi &= 2(1-z)\,.
\end{aligned}
$$

Outline

Simple cosmological model

- Homogeneous, isotropic FLRW universe.
- Matter content given by perfect fluid matter.
- Copernican principle: common evolution for all matter sectors.
  - ⇒ All metrics are equal: $g^I_{ab} = g_{ab}$.
  - ⇒ All energy-momentum tensors are equal: $T^I_{ab} = T_{ab}$.

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- PPN constraints on c_1 and c_2:

$$c_1 + c_2 = \frac{1}{1 + (N-1)z}\,.$$

$\Rightarrow$ Negative effective gravitational constant for $z = -1$ and $N > 2$.

Cosmological equations of motion

- Insert Robertson–Walker metric into equations of motion:

$$8\pi\rho = \frac{3}{2-N}\left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right),$$

$$8\pi p = -\frac{1}{2-N}\left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right).$$

$\Rightarrow$ Positive matter density $\rho > 0$ requires $k = -1$ and $\dot{a}^2 < 1$.

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- Acceleration equation:

$$\frac{\ddot{a}}{a} = \frac{4\pi(N-2)}{3}(\rho + 3p).$$

⇒ Acceleration must be positive for standard model matter.

Explicit solution

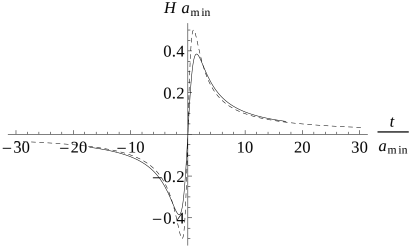
- Equation of state: $p = \omega\rho$; dust: $\omega = 0$, radiation: $\omega = 1/3$.
- General solution using conformal time η defined by $dt = a\,d\eta$:

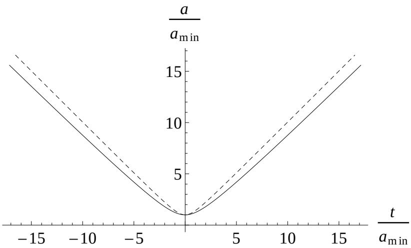
$$a = a_{\text{min}} \left(\cosh\left(\frac{3\omega+1}{2}(\eta-\eta_0)\right)\right)^{\frac{2}{3\omega+1}},$$

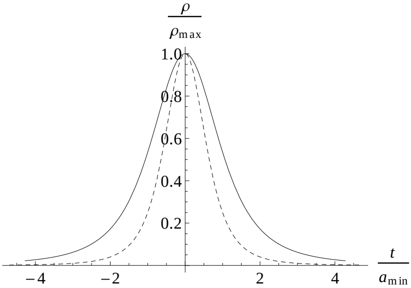
$$\rho = \rho_{\text{max}} \left(\cosh\left(\frac{3\omega+1}{2}(\eta-\eta_0)\right)\right)^{-\frac{6\omega+6}{3\omega+1}}.$$

$\Rightarrow$ Positive minimal radius a_{min}. [MH, M. Wohlfarth '10]

Cosmological evolution







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Summary

- Multimetric PPN formalism:
  - Assume perfect fluid matter.
  - Determine post-Newtonian metric.
  - PPN parameters $\alpha^{IJ}, \gamma^{IJ}, \theta^{IJ}, \sigma_{\pm}^{IJ}, \phi_{p}^{IJ}, \phi_{\Pi}^{IJ}, \omega_{1}^{IJ}, \omega_{2}^{IJ}, \psi_{1}^{IJK}, \ldots, \psi_{7}^{IJK}$.
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  - 13 parameters accessible to visible matter experiments.
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- Application to multimetric repulsive gravity:
  - Simple model dependent on 12 constant parameters.
  - 6 parameters fixed by experimental consistency.
  - Experimentally consistent model with 6 free parameters.
  - Relation between repulsive gravity and accelerating expansion.

Outlook

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  - Effects of new PPN potentials on current experiments?
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- Further experimental tests of multimetric gravity:
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  - Gravitational waves: parameterized post-Einsteinian formalism?
  - Cosmology: Cosmic microwave background?