Geodesic motion and the magnitude-redshift relation on cosmologically symmetric Finsler spacetimes

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- Modify spacetime geometry to address open problems:
  - Origin of dark matter and dark energy.
  - Homogeneity of the cosmic microwave background and inflation.
  - Fly-by anomaly in the solar system.

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  - Divide tangent spaces into space-, time-, lightlike vectors.
  - Provide notions of future and past.
  - Distinguish curves corresponding to physical trajectories.
  - Define proper time along physical trajectories.
  - Determine trajectories of freely falling test masses.
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  - Finsler geodesics determine notion of free fall.
  - Gravity theory on Finsler spacetimes exists.

Finsler geometry

- Clock postulate on metric spacetime: proper time is arc length

$$s[\gamma] = \int_{t_1}^{t_2} \sqrt{-g_{\mu\nu}(\gamma(t))\dot{\gamma}^{\mu}(t)\dot{\gamma}^{\nu}(t)}dt\,.$$

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- Generalized clock postulate on Finsler spacetimes:
  - General length functional:

$$s[\gamma] = \int_{t_1}^{t_2} F(\gamma(t), \dot{\gamma}(t))dt\,.$$

  - Finsler function $F : TM \to \mathbb{R}^+$.
  - Parametrization invariance requires homogeneity:

$$F(x, \lambda y) = \lambda F(x, y) \quad \forall \lambda > 0\,.$$

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- F is not differentiable on null structure $\Rightarrow$ use $L = F^h$ instead.

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  - Spherical coordinates t, r, ϑ, φ on M.
  - Coordinates y, u, v, w on each T_xM:

$$y\partial_t + w\left[\cos u\sqrt{1-kr^2}\partial_r + \frac{\sin u}{r}\left(\cos v\partial_\vartheta + \frac{\sin v}{\sin\vartheta}\partial_\varphi\right)\right] \in T_xM\,.$$

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- Cosmologically symmetric Finsler spacetime:
  - Symmetry under rotations and translations.
  - Most general geometry function: $L(t, y, w)$.
  - Homogeneity condition: $L(t, \lambda y, \lambda w) = \lambda L(t, y, w)$.
  - Express Finsler function as $L(t, y, w) = y^h\tilde{L}(t, w/y)$.

Constants of motion

- Construction of constants of motion:
  - L itself is always constant along any geodesic.
  - Symmetry generating vector field on M yields constant of motion:
    - Translations $\tau_i \Rightarrow$ linear momenta Π_i.
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- Constants of motion in cosmological symmetry:

$$C_0 = L = y^h \tilde{L}\,, \quad {C_4}^2 = \frac{\Lambda_3^2}{\vec{\Lambda}^2} = \sin^2 v \sin^2\theta\,,$$

$${C_1}^2 = \vec{\Pi}^2 + k\vec{\Lambda}^2 = y^{2h-2}\tilde{L}_w^2\,, \quad {C_2}^2 = \frac{\vec{\Lambda}^2}{\vec{\Pi}^2 + k\vec{\Lambda}^2} = r^2 \sin^2 u\,,$$

$$C_3 = -\arctan\frac{\Lambda_1}{\Lambda_2} = \phi + \arctan(\tan v \cos\theta)\,,$$

$$C_5 = \frac{\Pi_3}{C_1} = \sin u \cos v \sin\theta \sqrt{1 - kr^2} - \cos u \cos\theta\,,$$

$$C_6 = \frac{\Lambda_1\Pi_2 - \Pi_1\Lambda_2}{C_1^2 C_2} = \sin u \cos\theta \sqrt{1 - kr^2} + \cos u \cos v \sin\theta\,.$$

Geodesic motion

- Tangent $\dot{\gamma}(\lambda)$ of curve $\gamma(\lambda)$ in cosmological coordinates:

$$\dot{t} = y\,, \quad \dot{\varphi} = \frac{w \sin u \sin v}{r \sin\theta}\,,$$
$$\dot{r} = w\sqrt{1 - kr^2}\cos u\,, \quad \dot{\theta} = \frac{w \sin u \cos v}{r}\,.$$

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- Use constants of motion to determine remaining equations:

$$0 = \dot{C}_0 = y^{h-2}\left[y^3\tilde{L}_t + hy\tilde{L}\dot{y} - (w\dot{y} - y\dot{w})\tilde{L}_w\right]\,,$$
$$0 = \dot{C}_1 = y^{h-3}\left[y^3\tilde{L}_{tw} + (h-1)y\tilde{L}_w\dot{y} - (w\dot{y} - y\dot{w})\tilde{L}_{ww}\right]\,,$$
$$0 = \dot{C}_2 = \left(w \sin u\sqrt{1 - kr^2} + r\dot{u}\right)\cos u\,,$$
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$\Rightarrow$ Geodesics as integral curves of vector field **S** on *TM*.

Radial geodesics

- Purely radial motion:

$$t = t(\lambda)\,, \quad r = r(\lambda)\,, \quad \theta = \frac{\pi}{2}\,, \quad \phi = 0$$

$$\Rightarrow y = \dot{t}\,, \quad u = 0\,, \quad v = 0\,, \quad w\sqrt{1 - kr^2} = \dot{r}\,.$$

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- Lightlike radial geodesics:
  - Make use of $C_0 = L = y^h \tilde{L} \equiv 0$ along null geodesics.
  - Solve $0 = \tilde{L}(t, w/y)$ with $\dot{t} = y > 0$ for all $t \in \mathbb{R}$.
  - $\Rightarrow$ Solution $w/y = \mathring{W}(t)$ satisfies $\tilde{L}(t, \mathring{W}(t)) \equiv 0$.
  - Integrate to obtain solution for geodesic:

$$\frac{dr}{dt} = \mathring{W}(t)\sqrt{1 - kr^2} \Rightarrow \int_{r_e}^{r_o} \frac{dr}{\sqrt{1 - kr^2}} = \int_{t_e}^{t_o} \mathring{W}(t)\,dt\,.$$

Redshift of a light signal

- Propagation of two wave packets:
  - Source and observer and fixed co-moving coordinates r_e and r_o.
  - Wave packets emitted at times $t_{e,1}$ and $t_{e,2}$ from r_e.
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- Both packets travel identical coordinate distances:

$$0 = \int_{t_{e,2}}^{t_{o,2}} \mathring{W}(t)dt - \int_{t_{e,1}}^{t_{o,1}} \mathring{W}(t)dt \approx \mathring{W}(t_o)\Delta t_o - \mathring{W}(t_e)\Delta t_e \,.$$

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- Compare proper time period of emitted and observed signals:

$$1 + z = \frac{\Delta\tau_o}{\Delta\tau_e} = \left(\frac{|\mathring{L}(t_o)|}{|\mathring{L}(t_e)|}\right)^{\frac{1}{h}} \frac{\mathring{W}(t_e)}{\mathring{W}(t_o)} = \frac{\mathring{W}_L(t_e)}{\mathring{W}_L(t_o)} \,.$$

Magnitude of a distant source

- Ratio $\mathfrak{P}/\mathfrak{L}$ of received vs. emitted power:
  - Rate of photons decreased by factor $1+z$.
  - Energy of each photon decreased by factor $1+z$.

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  - Radial part of Finsler metric for co-moving receiver:

$$r^2 \tilde{L}^{\frac{2}{h}-1} \tilde{L}_{ww} \left(d\theta \otimes d\theta + \sin^2\theta \, d\phi \otimes d\phi\right) .$$

  - Surface area of illuminated sphere: $A = 4\pi r^2 \left|\tilde{L}^{\frac{2}{h}-1} \tilde{L}_{ww}\right|$.

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  - Surface area of illuminated sphere: $A = 4\pi r^2 \left|\tilde{L}^{\frac{2}{h}-1} \tilde{L}_{ww}\right|$.
- Magnitude derived from radiation flux:
  - Radiation flux $\mathfrak{S} = \frac{\mathfrak{P}}{A}$:

$$\mathfrak{S} = \frac{\mathfrak{L}}{4\pi r^2 (1+z)^2 \left|\tilde{L}^{\frac{2}{h}-1} \tilde{L}_{ww}\right|} .$$

  - Magnitude $m = -\frac{5}{2} \log_{10} \mathfrak{S} + \text{const.}$:

$$m = 5 \log_{10}[r(1+z)] + \frac{5}{2} \log_{10} \left|\tilde{L}^{\frac{2}{h}-1} \tilde{L}_{ww}\right| - \frac{5}{2} \log_{10} \mathfrak{L} + \text{const.}$$

Relating magnitude and redshift

- General procedure:
  - Consider fixed observation time t_o.
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- Magnitude-redshift relation:

$$m(z) = 5\log_{10} z + \frac{5}{2\ln 10}\left(2 + \frac{\mathring{W}_1 \mathring{W}_{L0}}{\mathring{W}_0 \mathring{W}_{L1}} - \frac{\mathring{W}_{L0} \mathring{W}_{L2}}{\mathring{W}_{L1}{}^2}\right) z + \mathcal{O}(z^2) - \frac{5}{2}\log_{10} \mathfrak{L} + \text{const.}$$

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- Magnitude-redshift relation:

$$m(z) = 5\log_{10} z + \frac{5}{2\ln 10}(1-q)\,z + \mathcal{O}(z^2) - \frac{5}{2}\log_{10}\mathfrak{L} + \text{const.}$$

- Deceleration parameter $q = \frac{\mathring{W}_{L0}\mathring{W}_{L2}}{\mathring{W}_{L1}^2} - \frac{\mathring{W}_1\mathring{W}_{L0}}{\mathring{W}_0\mathring{W}_{L1}} - 1$.

Example: FLRW metric spacetime

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- Conventional series expansion for scale factor:

$$a(t) = a_0\left[1 + H_0(t - t_o) - \frac{1}{2}H_0^2 q_0(t - t_o)^2\right] + \mathcal{O}\left((t - t_o)^3\right)\,.$$

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- Deceleration parameter: $q = q_0$.

Example: Bogoslovsky length measure

- Geometry function with $h = 4$:

$$L = \left(g_{ab}(x)y^a y^b\right)(A_c(x)y^c) = \left(-y^2 + a(t)^2 w^2\right) b(t)^2 \,.$$

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$$b(t) = b_0 + b_1(t - t_o) + \frac{1}{2}b_2(t - t_o)^2 + \mathcal{O}\left((t - t_o)^3\right) \,.$$

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$$b(t) = b_0 + b_1(t - t_o) + \frac{1}{2}b_2(t - t_o)^2 + \mathcal{O}\left((t - t_o)^3\right) \,.$$

- Deceleration parameter: $q = \frac{H_0^2 q_0 + \frac{1}{2}\left(\frac{b_1}{b_0}\right)^2 - H_0 \frac{1}{2}\frac{b_1}{b_0} - \frac{1}{2}\frac{b_2}{b_0}}{\left(H_0 + \frac{1}{2}\frac{b_1}{b_0}\right)^2}$.

Example: Randers length measure

- Geometry function with $h = 2$:

$$L = \left(\sqrt{|g_{ab}(x)y^a y^b|} + A_a(x)y^a\right)^2$$
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- Use same series expansion for parameter functions as before.
- Deceleration parameter in terms of series expansion:

$$q = q_0 - \frac{H_0(1 + 2q_0)b_1 + b_2}{H_0^2} + \mathcal{O}(b^2) .$$

Conclusion

- Summary:
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    - Based on 1-homogeneous Finsler length function $F = L^{\frac{1}{n}}$ on TM.
    - Make use of tensor fields on the tangent bundle TM.
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- Outlook:
  - Construct source term for gravitational field equations for fluids.
  - Solve cosmologically symmetric Finsler field equations.
  - Calculate further cosmological parameters (inflation, CMB?).
  - Derive constraints from cosmological observations.