Fluid dynamics on generalized geometric backgrounds

Manuel Hohmann

Teoreetilise Füüsika Labor
Füüsika Instituut
Tartu Ülikool

3. October 2014

Outline

Outline

Fluids are everywhere

- Perfect fluid:
  - No shear stress, no friction.
  - Characterized by density ρ and pressure p.
    - Dust, dark matter: $p = 0$.
    - Radiation: $p = \frac{1}{3}\rho$.
    - Dark energy: $p < -\frac{1}{3}\rho$.
  - Used in cosmology, parameterized post-Newtonian formalism...

Fluids are everywhere

- Perfect fluid:
  - No shear stress, no friction.
  - Characterized by density ρ and pressure p.
    - Dust, dark matter: $p = 0$.
    - Radiation: $p = \frac{1}{3}\rho$.
    - Dark energy: $p < -\frac{1}{3}\rho$.
  - Used in cosmology, parameterized post-Newtonian formalism...
- Collisionless fluid:
  - Model for dark matter.
  - Used in structure formation...

Fluids are everywhere

- Perfect fluid:
  - No shear stress, no friction.
  - Characterized by density ρ and pressure p.
    - Dust, dark matter: $p = 0$.
    - Radiation: $p = \frac{1}{3}\rho$.
    - Dark energy: $p < -\frac{1}{3}\rho$.
  - Used in cosmology, parameterized post-Newtonian formalism...
- Collisionless fluid:
  - Model for dark matter.
  - Used in structure formation...
- Maxwell-Boltzmann gas:
  - Collisions described by Boltzmann equation.
  - Used in structure formation, atmosphere dynamics...

Fluids are everywhere

- Perfect fluid:
  - No shear stress, no friction.
  - Characterized by density ρ and pressure p.
    - Dust, dark matter: $p = 0$.
    - Radiation: $p = \frac{1}{3}\rho$.
    - Dark energy: $p < -\frac{1}{3}\rho$.
  - Used in cosmology, parameterized post-Newtonian formalism...
- Collisionless fluid:
  - Model for dark matter.
  - Used in structure formation...
- Maxwell-Boltzmann gas:
  - Collisions described by Boltzmann equation.
  - Used in structure formation, atmosphere dynamics...
- Charged, multi-component gas:
  - Plasma, interacting gas including recombination / ionization.
  - Used in stellar dynamics, pre-CMB era models...

From spacetime to observer space

- Fluid dynamics naturally lift to tangent bundle:
  - Fluids conveniently modeled by particle dynamics (SPH...).
  - Physical fluids constituted by particles.
  - Particle trajectories lift to tangent bundle: $\gamma \rightsquigarrow (\gamma, \dot{\gamma})$.

  $\Rightarrow$ Dynamics on the tangent bundle described by first order ODE.

From spacetime to observer space

- Fluid dynamics naturally lift to tangent bundle:
  - Fluids conveniently modeled by particle dynamics (SPH...).
  - Physical fluids constituted by particles.
  - Particle trajectories lift to tangent bundle: $\gamma \rightsquigarrow (\gamma, \dot{\gamma})$.

  $\Rightarrow$ Dynamics on the tangent bundle described by first order ODE.
- Velocity dependence of physical measurements:
  - Physical observables are tensor components.
  - Measured tensor components depend on observer velocity.
  - Physical observer velocities are future unit timelike vectors.

  $\Rightarrow$ Observer space is space of physical velocities.

From spacetime to observer space

- Fluid dynamics naturally lift to tangent bundle:
  - Fluids conveniently modeled by particle dynamics (SPH. . .).
  - Physical fluids constituted by particles.
  - Particle trajectories lift to tangent bundle: $\gamma \rightsquigarrow (\gamma, \dot{\gamma})$.

  $\Rightarrow$ Dynamics on the tangent bundle described by first order ODE.
- Velocity dependence of physical measurements:
  - Physical observables are tensor components.
  - Measured tensor components depend on observer velocity.
  - Physical observer velocities are future unit timelike vectors.

  $\Rightarrow$ Observer space is space of physical velocities.
- Quantum gravity: possible non-tensorial observer dependence.
- Modified gravity theories may have more general observer spaces.

$\Rightarrow$ Physical observables become functions on observer space!

From spacetime to observer space

- Fluid dynamics naturally lift to tangent bundle:
  - Fluids conveniently modeled by particle dynamics (SPH. . .).
  - Physical fluids constituted by particles.
  - Particle trajectories lift to tangent bundle: $\gamma \rightsquigarrow (\gamma, \dot{\gamma})$.

  ⇒ Dynamics on the tangent bundle described by first order ODE.
- Velocity dependence of physical measurements:
  - Physical observables are tensor components.
  - Measured tensor components depend on observer velocity.
  - Physical observer velocities are future unit timelike vectors.

  ⇒ Observer space is space of physical velocities.
- Quantum gravity: possible non-tensorial observer dependence.
- Modified gravity theories may have more general observer spaces.

⇒ Physical observables become functions on observer space!

- Space of observers corresponds to particle tangent vectors.

⇒ Consider fluid dynamics on observer space!

Finsler spacetimes

- Finsler geometry of space widely used in physics:
  - Approaches to quantum gravity
  - Electrodynamics in anisotropic media
  - Modeling of astronomical data

Finsler spacetimes

- Finsler geometry of space widely used in physics:
  - Approaches to quantum gravity
  - Electrodynamics in anisotropic media
  - Modeling of astronomical data
- Finsler geometry generalizes Riemannian geometry:
  - Geometry described by Finsler function on the tangent bundle.
  - Finsler function measures length of tangent vectors.
  - Well-defined notions of connections, curvature, parallel transport...

Finsler spacetimes

- Finsler geometry of space widely used in physics:
  - Approaches to quantum gravity
  - Electrodynamics in anisotropic media
  - Modeling of astronomical data
- Finsler geometry generalizes Riemannian geometry:
  - Geometry described by Finsler function on the tangent bundle.
  - Finsler function measures length of tangent vectors.
  - Well-defined notions of connections, curvature, parallel transport...
- Finsler spacetimes are suitable backgrounds for:
  - Gravity
  - Electrodynamics
  - Other matter field theories

Finsler spacetimes

- Finsler geometry of space widely used in physics:
  - Approaches to quantum gravity
  - Electrodynamics in anisotropic media
  - Modeling of astronomical data
- Finsler geometry generalizes Riemannian geometry:
  - Geometry described by Finsler function on the tangent bundle.
  - Finsler function measures length of tangent vectors.
  - Well-defined notions of connections, curvature, parallel transport...
- Finsler spacetimes are suitable backgrounds for:
  - Gravity
  - Electrodynamics
  - Other matter field theories
- Possible explanations of yet unexplained phenomena:
  - Fly-by anomaly
  - Galaxy rotation curves
  - Accelerating expansion of the universe

Outline

1. Motivation
2. Finsler geometry and observer space
3. Fluids on observer space
4. Conclusion

The clock postulate

- Proper time along a curve in Lorentzian spacetime:

$$\tau = \int_{t_1}^{t_2} \sqrt{-g_{ab}(x(t))\dot{x}^a(t)\dot{x}^b(t)}dt\,.$$

The clock postulate

- Proper time along a curve in Lorentzian spacetime:

$$\tau = \int_{t_1}^{t_2} \sqrt{-g_{ab}(x(t))\dot{x}^a(t)\dot{x}^b(t)}dt\,.$$

- Finsler geometry: use a more general length functional:

$$\tau = \int_{t_1}^{t_2} F(x(t), \dot{x}(t))dt\,.$$

- Finsler function $F : TM \to \mathbb{R}^+$.
- Parametrization invariance requires homogeneity:

$$F(x, \lambda y) = \lambda F(x, y) \quad \forall \lambda > 0\,.$$

Finsler spacetimes

- Finsler geometries suitable for spacetimes exist. [C. Pfeifer, M. Wohlfarth '11]

⇒ Finsler metric with Lorentz signature:

$$g^F_{ab}(x,y) = \frac{1}{2}\frac{\partial}{\partial y^a}\frac{\partial}{\partial y^b}F^2(x,y)\,.$$

⇒ Notion of timelike, lightlike, spacelike tangent vectors.

Finsler spacetimes

- Finsler geometries suitable for spacetimes exist. [C. Pfeifer, M. Wohlfarth '11]

⇒ Finsler metric with Lorentz signature:

$$g^F_{ab}(x,y) = \frac{1}{2}\frac{\partial}{\partial y^a}\frac{\partial}{\partial y^b}F^2(x,y)\,.$$

⇒ Notion of timelike, lightlike, spacelike tangent vectors.

- Unit vectors $y \in T_xM$ defined by

$$F^2(x,y) = g^F_{ab}(x,y)y^a y^b = 1\,.$$

⇒ Set $\Omega_x \subset T_xM$ of unit timelike vectors at $x \in M$.

Finsler spacetimes

- Finsler geometries suitable for spacetimes exist. [C. Pfeifer, M. Wohlfarth '11]

$\Rightarrow$ Finsler metric with Lorentz signature:

$$g^F_{ab}(x, y) = \frac{1}{2} \frac{\partial}{\partial y^a} \frac{\partial}{\partial y^b} F^2(x, y) \, .$$

$\Rightarrow$ Notion of timelike, lightlike, spacelike tangent vectors.

- Unit vectors $y \in T_x M$ defined by

$$F^2(x, y) = g^F_{ab}(x, y) y^a y^b = 1 \, .$$

$\Rightarrow$ Set $\Omega_x \subset T_x M$ of unit timelike vectors at $x \in M$.

- Ω_x contains a closed connected component $S_x \subseteq \Omega_x$.

$\rightsquigarrow$ Causality: S_x corresponds to physical observers.

Geometry on the tangent bundle

- Cartan non-linear connection:

$$N^a{}_b = \frac{1}{4}\bar{\partial}_b\left[g^{F\,ac}(y^d\partial_d\bar{\partial}_c F^2 - \partial_c F^2)\right]$$

Geometry on the tangent bundle

- Cartan non-linear connection:

$$N^a{}_b = \frac{1}{4}\bar{\partial}_b \left[g^{F\,ac}(y^d \partial_d \bar{\partial}_c F^2 - \partial_c F^2) \right]$$

⇒ Split of the tangent and cotangent bundles:

  - Tangent bundle: $TTM = HTM \oplus VTM$

$$\delta_a = \partial_a - N^b{}_a \bar{\partial}_b \,, \quad \bar{\partial}_a$$

  - Cotangent bundle: $T^*TM = H^*TM \oplus V^*TM$

$$dx^a \,, \quad \delta y^a = dy^a + N^a{}_b dx^b$$

Geometry on the tangent bundle

- Cartan non-linear connection:

$$N^a{}_b = \frac{1}{4}\bar{\partial}_b\left[g^{F\,ac}(y^d\partial_d\bar{\partial}_c F^2 - \partial_c F^2)\right]$$

⇒ Split of the tangent and cotangent bundles:

- Tangent bundle: $TTM = HTM \oplus VTM$

$$\delta_a = \partial_a - N^b{}_a\bar{\partial}_b\,, \quad \bar{\partial}_a$$

- Cotangent bundle: $T^*TM = H^*TM \oplus V^*TM$

$$dx^a\,, \quad \delta y^a = dy^a + N^a{}_b dx^b$$

- Sasaki metric:

$$G = -g^F_{ab}\,dx^a \otimes dx^b - \frac{g^F_{ab}}{F^2}\,\delta y^a \otimes \delta y^b$$

Geometry on the tangent bundle

- Cartan non-linear connection:

$$N^a{}_b = \frac{1}{4}\bar{\partial}_b \left[g^{F\,ac}(y^d \partial_d \bar{\partial}_c F^2 - \partial_c F^2) \right]$$

⇒ Split of the tangent and cotangent bundles:

- Tangent bundle: $TTM = HTM \oplus VTM$

$$\delta_a = \partial_a - N^b{}_a \bar{\partial}_b\,, \quad \bar{\partial}_a$$

- Cotangent bundle: $T^*TM = H^*TM \oplus V^*TM$

$$dx^a\,, \quad \delta y^a = dy^a + N^a{}_b dx^b$$

- Sasaki metric:

$$G = -g^F_{ab}\, dx^a \otimes dx^b - \frac{g^F_{ab}}{F^2}\, \delta y^a \otimes \delta y^b$$

- Geodesic spray:

$$\mathbf{S} = y^a \delta_a$$

Geometry on observer space

- Recall from the definition of Finsler spacetimes:
  - Set $\Omega_x \subset T_xM$ of unit timelike vectors at $x \in M$.
  - Physical observers correspond to $S_x \subseteq \Omega_x$.
- Definition of observer space:

$$O = \bigcup_{x \in M} S_x \subset TM\,.$$

Geometry on observer space

- Recall from the definition of Finsler spacetimes:
  - Set $\Omega_x \subset T_xM$ of unit timelike vectors at $x \in M$.
  - Physical observers correspond to $S_x \subseteq \Omega_x$.
- Definition of observer space:

$$O = \bigcup_{x \in M} S_x \subset TM\,.$$

- Sasaki metric $\tilde{G}$ on O given by pullback of G to O.
- Volume form Σ of Sasaki metric $\tilde{G}$.
- Geodesic spray $\mathbf{S}$ restricts to Reeb vector field $\mathbf{r}$ on O.

Geometry on observer space

- Recall from the definition of Finsler spacetimes:
  - Set $\Omega_x \subset T_xM$ of unit timelike vectors at $x \in M$.
  - Physical observers correspond to $S_x \subseteq \Omega_x$.
- Definition of observer space:

$$O = \bigcup_{x \in M} S_x \subset TM\,.$$

- Sasaki metric $\tilde{G}$ on O given by pullback of G to O.
- Volume form Σ of Sasaki metric $\tilde{G}$.
- Geodesic spray $\mathbf{S}$ restricts to Reeb vector field $\mathbf{r}$ on O.
- Geodesic hypersurface measure $\omega = \iota_{\mathbf{r}}\Sigma$.
- Note that $\mathcal{L}_{\mathbf{r}}\Sigma = 0$ and $d\omega = 0$.

From metric to Finsler geometry

Tangent bundle geometry:

- Finsler function:

$$F(x,y) = \sqrt{|g_{ab}(x)y^a y^b|}$$

- Finsler metric:

$$g^F_{ab}(x,y) = \begin{cases} -g_{ab}(x) & y \text{ timelike} \\ g_{ab}(x) & y \text{ spacelike} \end{cases}$$

- Cartan non-linear connection:

$$N^a{}_b(x,y) = \Gamma^a{}_{bc}(x)y^c$$

From metric to Finsler geometry

Tangent bundle geometry:

- Finsler function:

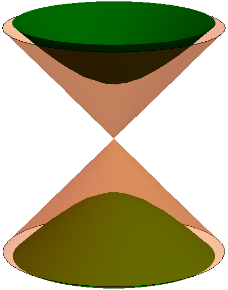
$$F(x, y) = \sqrt{|g_{ab}(x) y^a y^b|}$$

- Finsler metric:

$$g^F_{ab}(x, y) = \begin{cases} -g_{ab}(x) & y \text{ timelike} \\ g_{ab}(x) & y \text{ spacelike} \end{cases}$$

- Cartan non-linear connection:

$$N^a{}_b(x, y) = \Gamma^a{}_{bc}(x) y^c$$

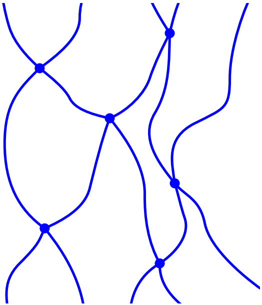


- Observer space:
  - Space Ω_x of unit timelike vectors at $x \in M$.
  - Space S_x of future unit timelike vectors at $x \in M$.
  - Observer space O: union of shells S_x.

Outline

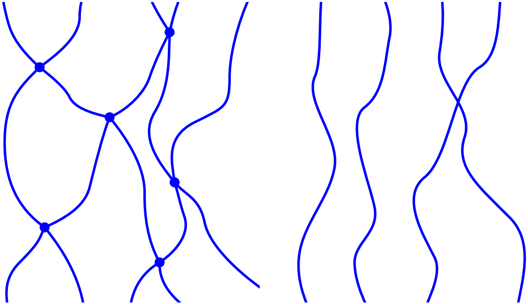
Definition of fluids

- Single-component fluid:
  - Constituted by classical, relativistic particles.
  - Particles have equal properties (mass, charge, . . .).
  - Particles follow piecewise geodesic curves.
  - Endpoints of geodesics are interactions with other particles.



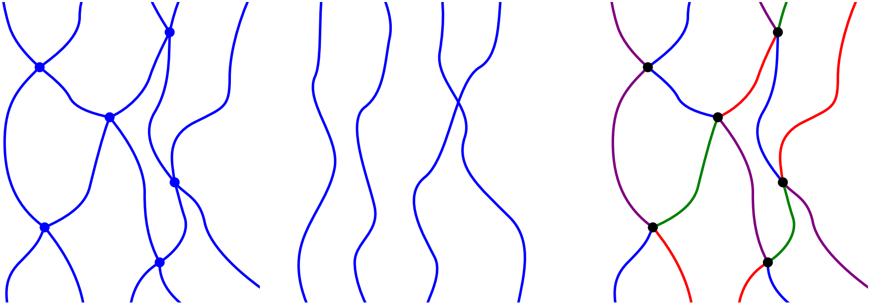
Definition of fluids

- Single-component fluid:
  - Constituted by classical, relativistic particles.
  - Particles have equal properties (mass, charge, . . .).
  - Particles follow piecewise geodesic curves.
  - Endpoints of geodesics are interactions with other particles.
- Collisionless fluid:
  - Particles do not interact with other particles.
  - $\Rightarrow$ Particles follow geodesics.



Definition of fluids

- Single-component fluid:
  - Constituted by classical, relativistic particles.
  - Particles have equal properties (mass, charge, ...).
  - Particles follow piecewise geodesic curves.
  - Endpoints of geodesics are interactions with other particles.
- Collisionless fluid:
  - Particles do not interact with other particles.
  - $\Rightarrow$ Particles follow geodesics.
- Multi-component fluid: multiple types of particles.



Geodesics on observer space

- Dynamics of fluids depends on geodesic equation.
- Geodesic equation for curve $x(\tau)$ on spacetime M:

$$\ddot{x}^a + N^a{}_b(x, \dot{x})\dot{x}^b = 0\,.$$

Geodesics on observer space

- Dynamics of fluids depends on geodesic equation.
- Geodesic equation for curve $x(\tau)$ on spacetime M:

$$\ddot{x}^a + N^a{}_b(x, \dot{x})\dot{x}^b = 0\,.$$

- Canonical lift of curve to tangent bundle TM:

$$x\,, \quad y = \dot{x}\,.$$

- Lift of geodesic equation:

$$\dot{x}^a = y^a\,, \quad \dot{y}^a = -N^a{}_b(x, y)y^b\,.$$

Geodesics on observer space

- Dynamics of fluids depends on geodesic equation.
- Geodesic equation for curve $x(\tau)$ on spacetime M:

$$\ddot{x}^a + N^a{}_b(x, \dot{x})\dot{x}^b = 0\,.$$

- Canonical lift of curve to tangent bundle TM:

$$x\,, \quad y = \dot{x}\,.$$

- Lift of geodesic equation:

$$\dot{x}^a = y^a\,, \quad \dot{y}^a = -N^a{}_b(x, y)y^b\,.$$

⇒ Solutions are integral curves of vector field:

$$y^a\partial_a - y^bN^a{}_b\bar{\partial}_a = y^a\delta_a = \mathbf{S}\,.$$

Geodesics on observer space

- Dynamics of fluids depends on geodesic equation.
- Geodesic equation for curve $x(\tau)$ on spacetime M:

$$\ddot{x}^a + N^a{}_b(x, \dot{x})\dot{x}^b = 0\,.$$

- Canonical lift of curve to tangent bundle TM:

$$x\,, \quad y = \dot{x}\,.$$

- Lift of geodesic equation:

$$\dot{x}^a = y^a\,, \quad \dot{y}^a = -N^a{}_b(x, y)y^b\,.$$

⇒ Solutions are integral curves of vector field:

$$y^a\partial_a - y^b N^a{}_b\bar{\partial}_a = y^a\delta_a = \mathbf{S}\,.$$

- Tangent vectors are future unit timelike: $(x, y) \in O$.

⇒ Particle trajectories are piecewise integral curves of **r** on O.

One-particle distribution function

- Recall: $\omega = \iota_{\mathbf{r}}\Sigma \in \Omega^6(O)$ unique 6-form such that:
  - ω non-degenerate on every hypersurface not tangent to $\mathbf{r}$.
  - $d\omega = 0$.

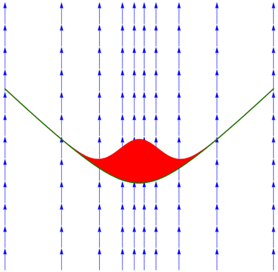
One-particle distribution function

- Recall: $\omega = \iota_{\mathbf{r}}\Sigma \in \Omega^6(O)$ unique 6-form such that:
  - ω non-degenerate on every hypersurface not tangent to **r**.
  - $d\omega = 0$.
- Define one-particle distribution function $\phi : O \to \mathbb{R}^+$ such that:

For every hypersurface $\sigma \subset O$,

$$N[\sigma] = \int_\sigma \phi\omega$$

# of particle trajectories through σ.



  -
  - Counting of particle trajectories respects hypersurface orientation.

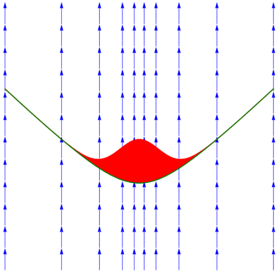
One-particle distribution function

- Recall: $\omega = \iota_{\mathbf{r}}\Sigma \in \Omega^6(O)$ unique 6-form such that:
  - ω non-degenerate on every hypersurface not tangent to $\mathbf{r}$.
  - $d\omega = 0$.
- Define one-particle distribution function $\phi : O \to \mathbb{R}^+$ such that:

For every hypersurface $\sigma \subset O$,

$$N[\sigma] = \int_\sigma \phi\omega$$

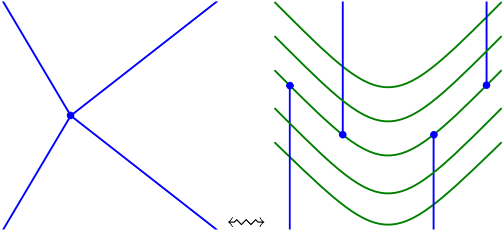
# of particle trajectories through σ.



  -
  - Counting of particle trajectories respects hypersurface orientation.
- For multi-component fluids: ϕ_i for each component i.

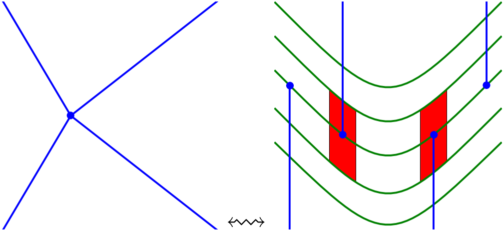
Collisions & the Liouville equation

- Collision in spacetime $\leftrightsquigarrow$ interruption in observer space.



Collisions & the Liouville equation

- Collision in spacetime $\leftrightsquigarrow$ interruption in observer space.



- For any open set $V \in O$,

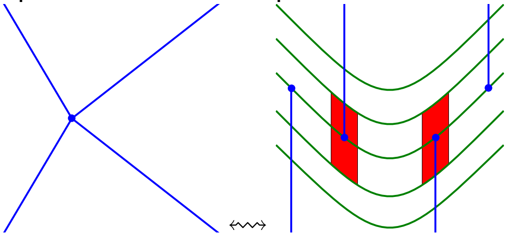
$$\int_{\partial V} \phi\omega = \int_V d(\phi\omega) = \int_V \mathcal{L}_{\mathbf{r}}\phi\Sigma$$

# of outbound trajectories - # of inbound trajectories.

$\Rightarrow$ Collision density measured by $\mathcal{L}_{\mathbf{r}}\phi$.

Collisions & the Liouville equation

- Collision in spacetime $\leftrightsquigarrow$ interruption in observer space.



- For any open set $V \in O$,

$$\int_{\partial V} \phi\omega = \int_V d(\phi\omega) = \int_V \mathcal{L}_{\mathbf{r}}\phi\Sigma$$

# of outbound trajectories - # of inbound trajectories.

$\Rightarrow$ Collision density measured by $\mathcal{L}_{\mathbf{r}}\phi$.

- Collisionless fluid: trajectories have no endpoints, $\mathcal{L}_{\mathbf{r}}\phi = 0$.

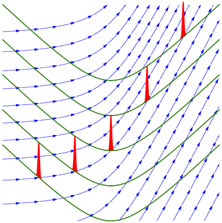
$\Rightarrow$ Simple, first order equation of motion for collisionless fluid.

$\Rightarrow$ ϕ is constant along integral curves of $\mathbf{r}$.

Examples of fluids

Geodesic dust fluid:

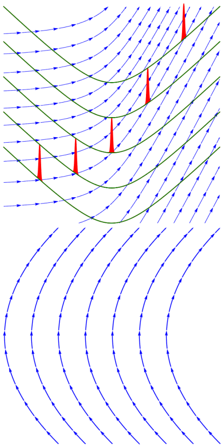
$\phi(x, y) \sim \delta(y - u(x))$.



Examples of fluids

Geodesic dust fluid:

$$\phi(x, y) \sim \delta(y - u(x))\,.$$

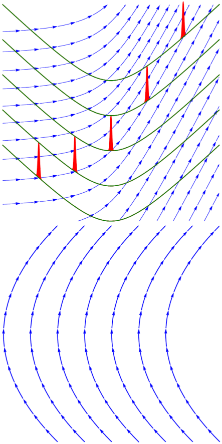


"Jenkka"

Examples of fluids

Geodesic dust fluid:

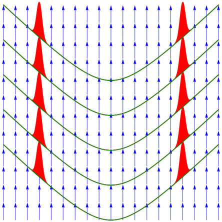
$$\phi(x, y) \sim \delta(y - u(x))\,.$$



"Jenkka"

Collisionless fluid:

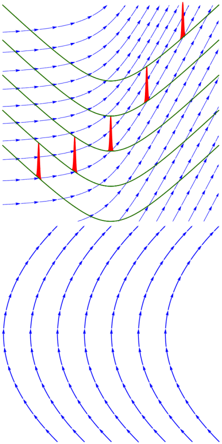
$$\mathcal{L}_{\mathbf{r}}\phi = 0\,.$$



Examples of fluids

Geodesic dust fluid:

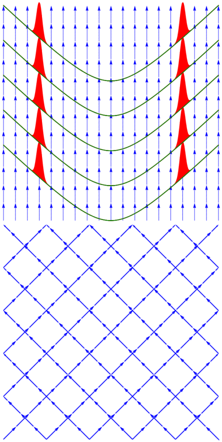
$$\phi(x, y) \sim \delta(y - u(x)) \, .$$



"Jenkka"

Collisionless fluid:

$$\mathcal{L}_{\mathbf{r}}\phi = 0 \, .$$

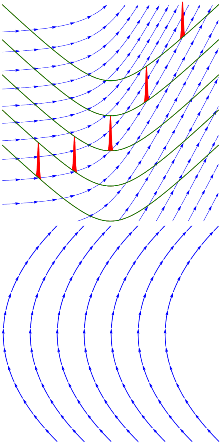


"Polkka"

Examples of fluids

Geodesic dust fluid:

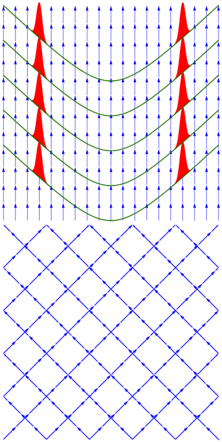
$$\phi(x, y) \sim \delta(y - u(x))\,.$$



"Jenkka"

Collisionless fluid:

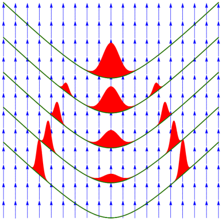
$$\mathcal{L}_{\mathbf{r}}\phi = 0\,.$$



"Polkka"

Interacting fluid:

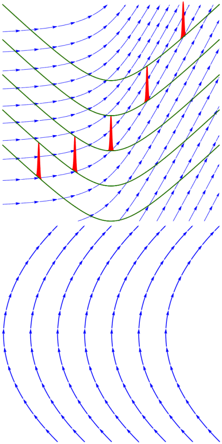
$$\mathcal{L}_{\mathbf{r}}\phi \neq 0\,.$$



Examples of fluids

Geodesic dust fluid:

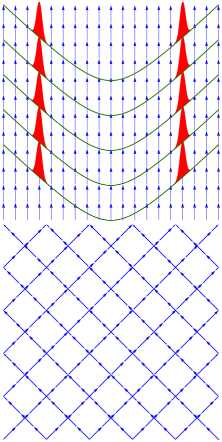
$$\phi(x,y) \sim \delta(y-u(x))\,.$$



"Jenkka"

Collisionless fluid:

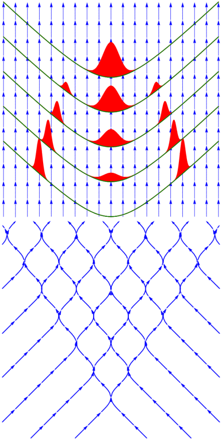
$$\mathcal{L}_{\mathbf{r}}\phi = 0\,.$$



"Polkka"

Interacting fluid:

$$\mathcal{L}_{\mathbf{r}}\phi \neq 0\,.$$



"Humppa"

Averaged quantities

- Volume form Π_x on unit timelike shells S_x induced by $\tilde{G}$.

Averaged quantities

- Volume form Π_x on unit timelike shells S_x induced by $\tilde{G}$.
- Averaged rest mass current density:

$$J^a(x) = m \int_{S_x} \phi y^a \Pi_x$$

- Averaged particle energy momentum tensor:

$$T^{ab}(x) = m \int_{S_x} \phi y^a y^b \Pi_x$$

Averaged quantities

- Volume form Π_x on unit timelike shells S_x induced by $\tilde{G}$.
- Averaged rest mass current density:

$$J^a(x) = m \int_{S_x} \phi y^a \Pi_x$$

- Averaged particle energy momentum tensor:

$$T^{ab}(x) = m \int_{S_x} \phi y^a y^b \Pi_x$$

$\Rightarrow$ Connection to well-known spacetime observables.

$\Rightarrow$ Connection to measurements.

Symmetric solutions

- Infinitesimal diffeomorphism described by vector field ξ on M.
- Canonical lift of ξ to vector field on TM:

$$\hat{\xi} = \xi^a \partial_a + y^a \partial_a \xi^b \bar{\partial}_b$$

Symmetric solutions

- Infinitesimal diffeomorphism described by vector field ξ on M.
- Canonical lift of ξ to vector field on TM:

$$\hat{\xi} = \xi^a \partial_a + y^a \partial_a \xi^b \bar{\partial}_b$$

- Killing vector field ξ:

$$\mathcal{L}_{\hat{\xi}} F = 0$$

$\Rightarrow$ $\hat{\xi}$ is tangent to observer space $O \subset TM$.

Symmetric solutions

- Infinitesimal diffeomorphism described by vector field ξ on M.
- Canonical lift of ξ to vector field on TM:

$$\hat{\xi} = \xi^a \partial_a + y^a \partial_a \xi^b \bar{\partial}_b$$

- Killing vector field ξ:

$$\mathcal{L}_{\hat{\xi}} F = 0$$

$\Rightarrow$ $\hat{\xi}$ is tangent to observer space $O \subset TM$.

$\Rightarrow$ Symmetric fluid solution:

$$\mathcal{L}_{\hat{\xi}} \phi = 0$$

Symmetric solutions

- Infinitesimal diffeomorphism described by vector field ξ on M.
- Canonical lift of ξ to vector field on TM:

$$\hat{\xi} = \xi^a \partial_a + y^a \partial_a \xi^b \bar{\partial}_b$$

- Killing vector field ξ:

$$\mathcal{L}_{\hat{\xi}} F = 0$$

$\Rightarrow$ $\hat{\xi}$ is tangent to observer space $O \subset TM$.

$\Rightarrow$ Symmetric fluid solution:

$$\mathcal{L}_{\hat{\xi}} \phi = 0$$

- Symmetry provides simplification of 7-dimensional O:
  - Spherical symmetry: 4 dimensions remain.
  - Static spherical symmetry: 3 dimensions remain.
  - Cosmological symmetry: 2 dimensions remain.

Outline

Summary

- Basic idea:
  - Model fluids by particle trajectories.
  - Lift trajectories from spacetime to observer space.
  - Describe geometry of observer space using Finsler geometry.

Summary

- Basic idea:
  - Model fluids by particle trajectories.
  - Lift trajectories from spacetime to observer space.
  - Describe geometry of observer space using Finsler geometry.
- Finsler geometry suitable for generalized gravity theories.

Summary

- Basic idea:
  - Model fluids by particle trajectories.
  - Lift trajectories from spacetime to observer space.
  - Describe geometry of observer space using Finsler geometry.
- Finsler geometry suitable for generalized gravity theories.
- Classical fluid variables obtained via averaging:
  - Particle flux density.
  - Energy-momentum tensor

Summary

- Basic idea:
  - Model fluids by particle trajectories.
  - Lift trajectories from spacetime to observer space.
  - Describe geometry of observer space using Finsler geometry.
- Finsler geometry suitable for generalized gravity theories.
- Classical fluid variables obtained via averaging:
  - Particle flux density.
  - Energy-momentum tensor
- Symmetries defined “as usual” by Killing vector fields.

Outlook

- Coupling of fluids to non-metric gravity theories.
- Cosmological solutions with non-metric geometry.
- Extension of parameterized post-Newtonian formalism.
- …

References

- Kinetic theory on the tangent bundle:

  - J. Ehlers, in: “General Relativity and Cosmology”, pp 1–70, Academic Press, New York / London, 1971.
  - O. Sarbach and T. Zannias, AIP Conf. Proc. **1548** (2013) 134 [arXiv:1303.2899 [gr-qc]].
  - O. Sarbach and T. Zannias, Class. Quant. Grav. **31** (2014) 085013 [arXiv:1309.2036 [gr-qc]].

- Finsler spacetimes:

  - C. Pfeifer and M. N. R. Wohlfarth, Phys. Rev. D **84** (2011) 044039 [arXiv:1104.1079 [gr-qc]].
  - C. Pfeifer and M. N. R. Wohlfarth, Phys. Rev. D **85** (2012) 064009 [arXiv:1112.5641 [gr-qc]].
  - MH, in: “Mathematical structures of the Universe”, pp 13–55, Copernicus Center Press, Krakow, 2014.

Kiitos!