# Finsler and Cartan geometric physical backgrounds

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# Outline


1. Introduction
2. Causality
3. Observers
4. Gravity
5. Conclusion

# Motivation

- Metric geometry of spacetime serves multiple roles:
  - Causality
  - Observers, observables and observations
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  - Spin foam networks
  - Causal dynamical triangulations

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⇒ More general, non-tensorial, "observer dependent" geometries:
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- How to serve the same roles as metric geometry?

# Why Finsler geometry of spacetimes?

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- Possible explanations of yet unexplained phenomena:
  - Fly-by anomaly
  - Galaxy rotation curves
  - Accelerating expansion of the universe

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- Problems:
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- Solution:
  - Consider space $O$ of all allowed observers.
  - Describe experiments on observer space instead of spacetime.
  - ⇒ Observer dependence of physical quantities follows naturally.
  - ⇒ No preferred observers.
  - Geometry of observer space modeled by Cartan geometry.

# Geometrical structures

## Metric geometry

Manifold $M$

Lorentzian metric $g$

Orientation

Time orientation

## Finsler geometry

Tangent bundle $TM$

Geometry function $L : TM \to \mathbb{R}$

Finsler function $F : TM \to \mathbb{R}$

Finsler metric $g^F(x, y)$

Cartan non-linear connection $N^a{}_b$

Cartan linear connection $\nabla$

## Cartan geometry

Lie group $G = \mathrm{ISO}_0(3, 1)$

Closed subgroup $K = \mathrm{SO}(3)$

Principal $K$-bundle $\pi : P \to O$

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## From metric to Finsler

Coordinates $(x^a)$ on $M$

Coordinates $(x^a, y^a)$ on $TM$

Define $L(x, y) = g_{ab}(x)y^a y^b$

## From Finsler to Cartan

Space $O$ of observer 4-velocities

Space $P$ of observer frames

Define $A$ from connection $\nabla$

# Metric spacetime geometry

- Ingredients of metric spacetime geometry:
  - 4-dimensional spacetime manifold $M$.
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- Clock postulate: proper time measured by arc length.

⇒ Arc length for curves $t \mapsto \gamma(t) \in M$ defined by the metric:

$$\tau_2 - \tau_1 = \int_{t_1}^{t_2} \sqrt{|g_{ab}(\gamma(t))\dot{\gamma}^a(t)\dot{\gamma}^b(t)|}dt\,.$$

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- Observables are components of tensor fields.
- Tensor components must be expressed in suitable basis.

⇒ Metric provides notion of orthonormal frames:

$$g_{ab}f_i^a f_j^b = \eta_{ij}\,.$$

# Basics of Finsler spacetimes

- Finsler geometry defined by length functional for curve $\gamma$:

$$\tau_2 - \tau_1 = \int_{t_1}^{t_2} F(\gamma(t), \dot{\gamma}(t))dt$$

- Finsler function $F : TM \to \mathbb{R}^+$.
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- Introduce manifold-induced coordinates $(x^a, y^a)$ on $TM$:
  - Coordinates $x^a$ on $M$.
  - Define coordinates $y^a$ for $y^a \frac{\partial}{\partial x^a} \in T_xM$.
  - Tangent bundle $TTM$ spanned by $\left\{\partial_a = \frac{\partial}{\partial x^a}, \bar{\partial}_a = \frac{\partial}{\partial y^a}\right\}$.

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- $n$-homogeneous functions on $TM$: $f(x, \lambda y) = \lambda^n f(x, y)$.
  - $n$-homogeneous smooth geometry function $L : TM \to \mathbb{R}$.
  - $\Rightarrow$ 1-homogeneous Finsler function $F = |L|^{\frac{1}{n}}$.

$\Rightarrow$ Finsler metric with Lorentz signature:

$$g^F_{ab}(x, y) = \frac{1}{2} \bar{\partial}_a \bar{\partial}_b F^2(x, y)\,.$$

# Connections on Finsler spacetimes

- Cartan non-linear connection:

$$N^a{}_b = \frac{1}{4}\bar{\partial}_b\left[g^{F\,ac}(y^d\partial_d\bar{\partial}_c F^2 - \partial_c F^2)\right]\,.$$

$\Rightarrow$ Berwald basis of $TTM$:

$$\{\delta_a = \partial_a - N^b{}_a\bar{\partial}_b, \bar{\partial}_a\}\,.$$

$\Rightarrow$ Dual Berwald basis of $T^*TM$:

$$\{dx^a, \delta y^a = dy^a + N^a{}_b dx^b\}\,.$$

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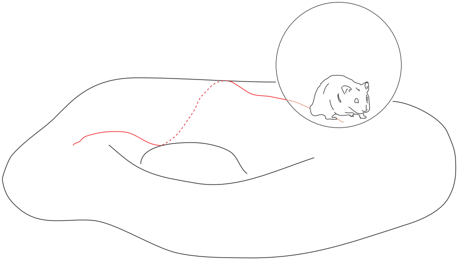
- Cartan linear connection:

$$\nabla_{\delta_a}\delta_b = F^c{}_{ab}\delta_c\,,\ \nabla_{\delta_a}\bar{\partial}_b = F^c{}_{ab}\bar{\partial}_c\,,\ \nabla_{\bar{\partial}_a}\delta_b = C^c{}_{ab}\delta_c\,,\ \nabla_{\bar{\partial}_a}\bar{\partial}_b = C^c{}_{ab}\bar{\partial}_c\,,$$

$$F^c{}_{ab} = \frac{1}{2}g^{F\,cd}(\delta_a g^F_{bd} + \delta_b g^F_{ad} - \delta_d g^F_{ab})\,,$$

$$C^c{}_{ab} = \frac{1}{2}g^{F\,cd}(\bar{\partial}_a g^F_{bd} + \bar{\partial}_b g^F_{ad} - \bar{\partial}_d g^F_{ab})\,.$$

# Toy model for Cartan geometry: The hamster ball



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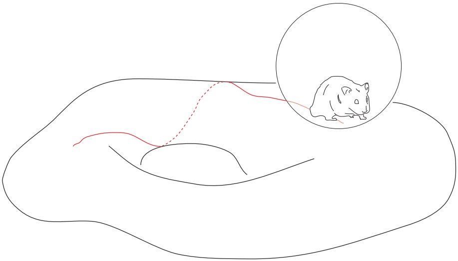

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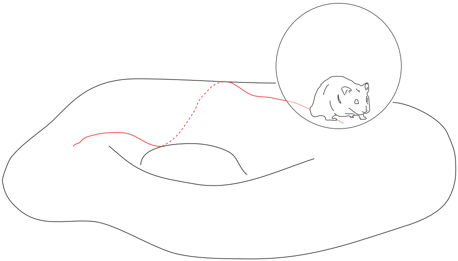

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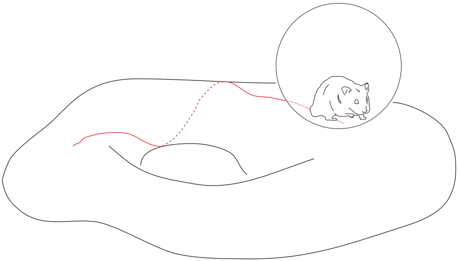

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$\Rightarrow$ Surface $M$ "traced" by $S^2 \cong \mathrm{SO}(3)/\mathrm{SO}(2) = G/H$.

$\Rightarrow$ Geometry of $M$ fully described by Hamster ball motion.

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# From metric to Finsler

- Metric-induced 2-homogeneous geometry function:

$$L(x, y) = g_{ab}(x) y^a y^b \,.$$

$\Rightarrow$ Finsler function $F(x, y) = \sqrt{|L(x, y)|}$.

$\Rightarrow$ Finsler metric

$$g^F(x, y) = \begin{cases} -g(x, y) & \text{for } y \text{ timelike,} \\ g(x, y) & \text{for } y \text{ spacelike.} \end{cases}$$

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⇒ Cartan linear connection:

$$F^a{}_{bc} = \Gamma^a{}_{bc}\,, \quad C^a{}_{bc} = 0\,.$$

# From Finsler to Cartan

- Need to construct $A \in \Omega^1(P, \mathfrak{g})$.
- Recall that

$$\begin{array}{ccccc} \mathfrak{g} & = & \mathfrak{h} & \oplus & \mathfrak{z} \\ A & = & \omega & + & e \end{array}$$

- Definition of $e$: Use the *solder form*:

$$e^i = {f^{-1}}^i{}_a dx^a \,.$$

- Definition of $\omega$: Use the *Cartan linear connection*:

$$\omega^i{}_j = {f^{-1}}^i{}_a \left[ df^a_j + f^b_j \left( dx^c F^a{}_{bc} + (dx^d N^c{}_d + df^c_0) C^a{}_{bc} \right) \right] \,.$$

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- Let $a = z^i \mathcal{Z}_i + \frac{1}{2} h^i{}_j \mathcal{H}_i{}^j \in \mathfrak{g}$.
- Fundamental vector fields:

$$\underline{A}(a) = z^i f_i^a \left(\partial_a - f_j^b F^c{}_{ab} \bar{\partial}_c^j\right) + \left(h^i{}_j f_i^a - h^i{}_0 f_i^b f_j^c C^a{}_{bc}\right) \bar{\partial}_a^j \,.$$

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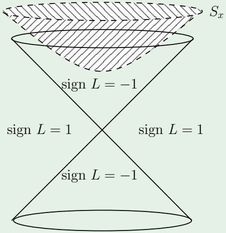

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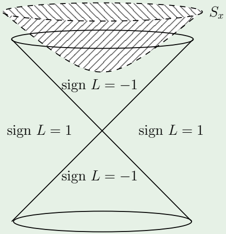


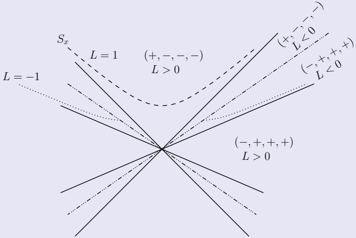

## Finsler geometry

Fundamental geometry function $L$

Hessian:

$$g^L_{ab}(x, y) = \frac{1}{2}\bar{\partial}_a\bar{\partial}_b L(x, y)$$

Use sign of $L$ and signature of $g^L$.



# Causal structure

## Metric geometry

Geometry function:

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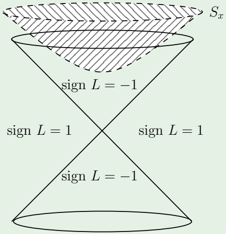


## Cartan geometry

Observer space:

$$O = \bigcup_{x \in M} S_x$$

$O$ contains only future unit timelike vectors.

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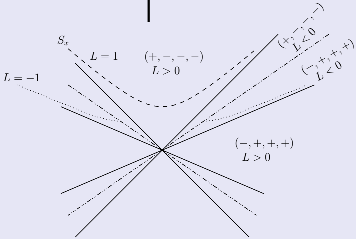

# Causality of Finsler spacetimes

- "Unit timelike condition" required for Finsler spacetimes:
  For all $x \in M$ the set

$$\Omega_x = \left\{ y \in T_x M \,\middle|\, |L(x,y)| = 1, \operatorname{sig} \bar{\partial}_a \bar{\partial}_b L(x,y) = (\epsilon, -\epsilon, -\epsilon, -\epsilon) \right\}$$

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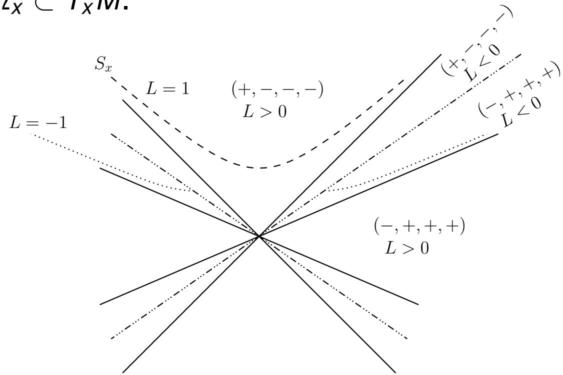
with $\epsilon = L(x,y)/|L(x,y)|$ contains a non-empty closed connected component $S_x \subseteq \Omega_x \subset T_xM$.

$\Rightarrow$ $S_x$ contains physical observers.

$\Rightarrow$ $\mathbb{R}^+ S_x$ is convex cone.



# The observer frame bundle

- Observer space of a Finsler spacetime:
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- Construct orthonormal observer frames:

  $\Rightarrow$ Complete $y = f_0$ to a frame $f_i$ with $g^F_{ab}(x, y) f_i^a f_j^b = -\eta_{ij}$.
  - Let $P$ be the space of all observer frames.
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  - Let $P$ be the space of all observer frames.
  - Natural projection $\pi : P \to O$ discards spatial frame components.
- Group action on the frame bundle:
  - SO(3) acts on spatial frame components by rotations.
  - Action is free and transitive on fibers of $\pi : P \to O$.

  $\Rightarrow$ $\pi : P \to O$ is principal $K$-bundle.

# Outline


1 Introduction

2 Causality

3 Observers

4 Gravity

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# Observers

## Metric geometry

Timelike curve $\gamma$:

$$\begin{array}{cccc} \gamma & : & \mathbb{R} & \to & M \\ & & \tau & \mapsto & \gamma(\tau) \end{array}$$

$$g_{ab}\dot{\gamma}^a\dot{\gamma}^b = -1$$

Orthonormal frame $f$:

$$f_0^a = \dot{\gamma}^a$$

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$$\Gamma(\tau) = (\gamma(\tau), \dot{\gamma}(\tau))$$

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Lift condition:

$$\tilde{e}^i\dot{\Gamma}(\tau) = \delta_0^i$$

Orthonormal frame $f$:

$$f \in \pi^{-1}(\Gamma(\tau)) \subset P$$

# Inertial observers

## Metric geometry

Minimize arc length integral:

$$\int_{t_1}^{t_2} \sqrt{|g_{ab}(\gamma(t))\dot{\gamma}^a(t)\dot{\gamma}^b(t)|}dt$$

Geodesic equation:

$$\ddot{\gamma}^a + \Gamma^a{}_{bc}\dot{\gamma}^b\dot{\gamma}^c = 0$$

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$\Rightarrow$ $\Gamma$ is integral curve of geodesic spray:

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- Observer curves:
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- Boost component of the tangent vector:
  - Measures acceleration in observer's frame.
  - Inertial observers are non-accelerating: $b^\alpha \dot{\Gamma} = 0$.
  - $\Rightarrow$ Inertial observers follow integral curves of time translation: $\dot{\Gamma} = \underline{e}_0$.

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1. Introduction
2. Causality
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# Gravity

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Einstein-Hilbert action:

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Using horizontal vector fields:

$$S_{\mathrm{H}} = \int_O \tilde{b}^\alpha([\underline{\tilde{e}}_\alpha, \underline{\tilde{e}}_0])\,\mathrm{Vol}_O$$

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# Gravity from Cartan to Finsler

- MacDowell-Mansouri gravity on observer space: [S. Gielen, D. Wise '12]

$$S_G = \int_O \epsilon_{\alpha\beta\gamma} \operatorname{tr}_{\mathfrak{h}}(F_{\mathfrak{h}} \wedge \star F_{\mathfrak{h}}) \wedge b^\alpha \wedge b^\beta \wedge b^\gamma$$

  - Hodge operator $\star$ on $\mathfrak{h}$.
  - Non-degenerate $H$-invariant inner product $\operatorname{tr}_{\mathfrak{h}}$ on $\mathfrak{h}$.
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- Translate terms into Finsler language (with $R = d\omega + \frac{1}{2}[\omega, \omega]$):
  - Curvature scalar:

$$[e, e] \wedge \star R \rightsquigarrow g^{F\,ab} R^c{}_{acb}\, dV\,.$$

  - Cosmological constant:

$$[e, e] \wedge \star [e, e] \rightsquigarrow dV\,.$$

  - Gauss-Bonnet term:

$$R \wedge \star R \rightsquigarrow \epsilon^{abcd} \epsilon^{efgh} R_{abef} R_{cdgh}\, dV\,.$$

$\Rightarrow$ Gravity theory on Finsler spacetime.

# Gravity from Finsler to Cartan

- Finsler gravity action: [C. Pfeifer, M. Wohlfarth '11]

$$S_G = \int_O d^4x\, d^3y \sqrt{-\tilde{G}} R^a{}_{ab} y^b \,.$$

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$$d^4x\, d^3y\, \sqrt{-\tilde{G}} = \epsilon_{ijkl}\epsilon_{\alpha\beta\gamma}\, e^i \wedge e^j \wedge e^k \wedge e^l \wedge b^\alpha \wedge b^\beta \wedge b^\gamma \,,$$
$$R^a{}_{ab} y^b = b^\alpha[\underline{A}(\mathcal{Z}_\alpha), \underline{A}(\mathcal{Z}_0)] \,.$$

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# Summary

- Finsler spacetimes
  - Generalization of metric spacetimes.
  - Geometry defined by function $L$ on $TM$.
  - Lengths measured by Finsler function $F = |L|^{\frac{1}{n}}$.
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- Different geometries provide compatible definitions of:
  - Causality
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# Open questions

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