Parametrized post-Newtonian limit of ghost-free bimetric massive gravity

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Motivation

- Not yet understood cosmological observations:
  - Homogeneity of cosmic microwave background.
  - Accelerating expansion of the universe.
  - Motion of galaxies and rotation curves.
  - Galactic mergers (Abell 520).

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- Solar system consistency?
  - Within Vainshtein radius: same as general relativity.
  - Beyond Vainshtein radius?

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$$- m^4 \sqrt{-\det g} \sum_{n=0}^{4} \beta_n e_n \left(\sqrt{g^{-1} f} \right)$$
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- Terms in the action:
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  - interaction potential
  - matter coupling
- Parameters: masses m_g, m_f, m; couplings $\beta_0, \ldots, \beta_4$.

Field equations

- Field equations:

$$m_g^2\left(R^g_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R^g\right) + m^4 V^g_{\mu\nu} = T^g_{\mu\nu}\,,$$

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- Energy-momentum tensors:

$$T^g_{\mu\nu} = -\frac{2}{\sqrt{-\det g}}\frac{\delta\left(\sqrt{-\det g}\mathcal{L}^g_m(g,\Phi_g)\right)}{\delta g^{\mu\nu}}\,,$$
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- Interaction terms:

$$V^g_{\mu\nu} = g_{\mu\rho}\sum_{n=0}^{3}(-1)^n\beta_n Y_n{}^\rho{}_\nu(A)\,, \quad V^f_{\mu\nu} = f_{\mu\rho}\sum_{n=0}^{3}(-1)^n\beta_{4-n} Y_n{}^\rho{}_\nu(A^{-1})\,.$$

Unique ghost-free interaction

- Matrix square root:

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- Explicit form of matrix invariants:
$$e_1(A) = \operatorname{tr} A\,, \quad e_2(A) = \frac{1}{2}\left[(\operatorname{tr} A)^2 - \operatorname{tr} A^2\right]\,,$$
$$e_0(A) = 1\,, \quad e_3(A) = \frac{1}{6}\left[(\operatorname{tr} A)^3 - 3\operatorname{tr} A\operatorname{tr} A^2 + 2\operatorname{tr} A^3\right]\,,$$
$$e_4(A) = \frac{1}{24}\left[(\operatorname{tr} A)^4 - 6(\operatorname{tr} A)^2\operatorname{tr} A^2 + 3(\operatorname{tr} A^2)^2 + 8\operatorname{tr} A\operatorname{tr} A^3 - 6\operatorname{tr} A^4\right]\,.$$

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- Interaction terms appearing in the field equations:

$$Y_n(A) = \sum_{k=0}^{n}(-1)^k e_k(A)A^{n-k}\,.$$

Flat, proportional background solution

- Assume existence of vacuum solution with $c > 0$:

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$\Rightarrow$ Consider only models which satisfy

$$\tilde{\beta}_0 = -3\tilde{\beta}_1 - 3\tilde{\beta}_2 - \tilde{\beta}_3\,, \quad \tilde{\beta}_4 = -\tilde{\beta}_1 - 3\tilde{\beta}_2 - 3\tilde{\beta}_3\,.$$

$\Rightarrow$ New free parameter $c > 0$ instead of β_0, β_4 in the action.

Static point mass source

- Energy-momentum tensors for perfect fluid:

$$T^{g\,\mu\nu} = (\rho^g + \rho^g\Pi^g + p^g)u^{g\,\mu}u^{g\,\nu} + p^g g^{\mu\nu}\,,$$
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$$\rho^g = M^g\delta(\vec{x})\,, \qquad \Pi^g = 0\,, \qquad p^g = 0\,, \qquad u^g \sim \partial_t\,,$$
$$\rho^f = M^f\frac{\delta(\vec{x})}{c^3}\,, \qquad \Pi^f = 0\,, \qquad p^f = 0\,, \qquad u^f \sim \partial_t\,.$$

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- Rescaling of mass parameters:

$$\tilde{m}_g = m_g\,, \quad \tilde{m}_f = cm_f \quad \tilde{M}^g = M^g\,, \quad \tilde{M}^f = cM^f\,.$$

Post-Newtonian metric ansatz

- PPN metric ansatz:

$$\begin{aligned}
g_{00} &= -1 + 2\frac{\tilde{\alpha}^{gg}\tilde{M}^g + \tilde{\alpha}^{gf}\tilde{M}^f}{r}\,,\\
g_{ij} &= \delta_{ij} + 2\frac{\tilde{\gamma}^{gg}\tilde{M}^g + \tilde{\gamma}^{gf}\tilde{M}^f}{r}\delta_{ij} + 2\frac{\tilde{\theta}^{gg}\tilde{M}^g + \tilde{\theta}^{gf}\tilde{M}^f}{r^3}x_ix_j\,,\\
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- PPN parameters (in general depend on interaction distance r):
  - $\tilde{\alpha}^{gg}, \tilde{\alpha}^{gf}, \tilde{\alpha}^{fg}, \tilde{\alpha}^{ff}$: Newtonian interaction.

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  - $\tilde{\theta}^{gg}, \tilde{\theta}^{gf}, \tilde{\theta}^{fg}, \tilde{\theta}^{ff}$: Off-diagonal contribution.
- Gauge choice: $\tilde{\theta}^{gg} = \tilde{\theta}^{ff} = 0$.

PPN parameters

- Solution for PPN parameters:

$$\tilde{\alpha}^{gg} = \frac{3\tilde{m}_g^2 + 4\tilde{m}_f^2 e^{-\mu r}}{24\pi\tilde{m}_g^2(\tilde{m}_f^2 + \tilde{m}_g^2)}, \quad \tilde{\alpha}^{ff} = \frac{3\tilde{m}_f^2 + 4\tilde{m}_g^2 e^{-\mu r}}{24\pi\tilde{m}_f^2(\tilde{m}_f^2 + \tilde{m}_g^2)},$$

$$\tilde{\alpha}^{gf} = \frac{3 - 4e^{-\mu r}}{24\pi(\tilde{m}_f^2 + \tilde{m}_g^2)}, \quad \tilde{\alpha}^{fg} = \frac{3 - 4e^{-\mu r}}{24\pi(\tilde{m}_f^2 + \tilde{m}_g^2)},$$

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$$\tilde{\gamma}^{gf} = \frac{9\tilde{m}_f^2 + 2(\tilde{m}_g^2 - 2\tilde{m}_f^2)e^{-\mu r}}{72\pi\tilde{m}_f^2(\tilde{m}_f^2 + \tilde{m}_g^2)} - \frac{\mu r(\mu r + 3) + 3}{36\pi\tilde{m}_f^2\mu^2 r^2}e^{-\mu r},$$

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$$\tilde{\theta}^{gf} = \frac{\mu r(\mu r + 3) + 3}{12\pi\tilde{m}_f^2\mu^2 r^2}e^{-\mu r}, \quad \tilde{\theta}^{fg} = \frac{\mu r(\mu r + 3) + 3}{12\pi\tilde{m}_g^2\mu^2 r^2}e^{-\mu r}.$$

Physical interpretation

- Physical meaning of PPN parameters:

	Newtonian gravity	light deflection
by visible matter	$\tilde{\alpha}^{gg} = \frac{3\tilde{m}_g{}^2 + 4\tilde{m}_f{}^2 e^{-\mu r}}{24\pi\tilde{m}_g{}^2(\tilde{m}_f{}^2 + \tilde{m}_g{}^2)}$	$\frac{\tilde{\gamma}^{gg}}{\tilde{\alpha}^{gg}} = \frac{3\tilde{m}_g{}^2 + 2\tilde{m}_f{}^2 e^{-\mu r}}{3\tilde{m}_g{}^2 + 4\tilde{m}_f{}^2 e^{-\mu r}}$
by dark matter	$\tilde{\alpha}^{gf} = \frac{3 - 4e^{-\mu r}}{24\pi(\tilde{m}_f{}^2 + \tilde{m}_g{}^2)}$	$\frac{\tilde{\gamma}^{gf} + \tilde{\theta}^{gf}/3}{\tilde{\alpha}^{gf}} = 1 + \frac{2(\tilde{m}_g{}^2 + 4\tilde{m}_f{}^2)}{3\tilde{m}_f{}^2(3e^{\mu r} - 4)}$

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- Constants appearing in PPN parameters:
  - Graviton mass:

$$\mu = m^2\sqrt{\left(\tilde{\beta}_1 + 2\tilde{\beta}_2 + \tilde{\beta}_3\right)\left(\frac{1}{\tilde{m}_f{}^2} + \frac{1}{\tilde{m}_g{}^2}\right)}\,.$$

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	Newtonian gravity	light deflection
by visible matter	$\tilde{\alpha}^{gg} = \frac{3\tilde{m}_g{}^2+4\tilde{m}_f{}^2e^{-\mu r}}{24\pi\tilde{m}_g{}^2(\tilde{m}_f{}^2+\tilde{m}_g{}^2)}$	$\frac{\tilde{\gamma}^{gg}}{\tilde{\alpha}^{gg}} = \frac{3\tilde{m}_g{}^2+2\tilde{m}_f{}^2e^{-\mu r}}{3\tilde{m}_g{}^2+4\tilde{m}_f{}^2e^{-\mu r}}$
by dark matter	$\tilde{\alpha}^{gf} = \frac{3-4e^{-\mu r}}{24\pi(\tilde{m}_f{}^2+\tilde{m}_g{}^2)}$	$\frac{\tilde{\gamma}^{gf}+\tilde{\theta}^{gf}/3}{\tilde{\alpha}^{gf}} = 1 + \frac{2(\tilde{m}_g{}^2+4\tilde{m}_f{}^2)}{3\tilde{m}_f{}^2(3e^{\mu r}-4)}$

- Constants appearing in PPN parameters:
  - Graviton mass:

$$\mu = m^2\sqrt{\left(\tilde{\beta}_1 + 2\tilde{\beta}_2 + \tilde{\beta}_3\right)\left(\frac{1}{\tilde{m}_f{}^2} + \frac{1}{\tilde{m}_g{}^2}\right)}.$$

  - Effective Planck masses $\tilde{m}_g$, $\tilde{m}_f$.

Solar system consistency

- Cassini tracking experiment (Shapiro delay by the sun):
  - Effective interaction distance: $r_0 \approx 1.6 R_\odot \approx 7.44 \cdot 10^{-3}\text{AU}$.
  - Measured PPN parameter: $\frac{\tilde{\gamma}^{gg}}{\tilde{\alpha}^{gg}} - 1 = (2.1 \pm 2.3) \cdot 10^{-5}$.

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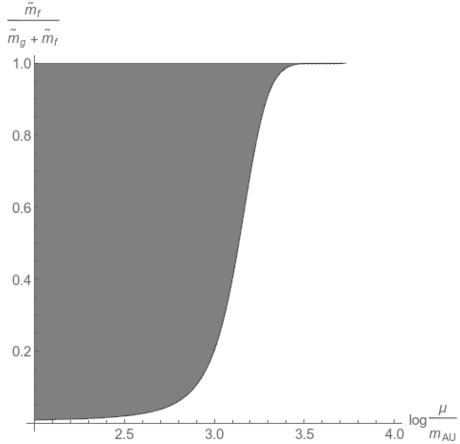
- Gray area excluded at 2σ (with $m_{\text{AU}} = 1\text{AU}^{-1} \approx 1.32 \cdot 10^{-18} \frac{\text{eV}}{c^2}$):



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- Outlook:
  - Include Vainshtein mechanism in analysis.
  - Calculate light deflection by massive graviton dark matter.
  - Extend analysis to PPN parameter β.
  - Consider more general theory with more metrics.