Teleparallel Geometry and Invariance of field equations

Christian Pfeifer
Center of Applied Space Technology and Microgravity, ZARM
University of Bremen

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1. Teleparallel/Weitzenböck Geometry

2. Symmetry and Covariance in teleparallel theories of Gravity

3. Conclusion

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2. Symmetry and Covariance in teleparallel theories of Gravity

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Geometric fields - General Connections - Weitzenböck Connection & Gauge

Geometry of a spacetime manifold M

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Affine connection and metric invariant under local Lorentz transformations (behave tensorial when index is changed)

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Weitzenböck Gauge: Choose $\Lambda^a{}_b = (\bar\Lambda^{-1})^a{}_b \Rightarrow \hat\Lambda^a{}_b = \delta^a_b \Rightarrow \Gamma^\mu{}_{\nu\rho}[\theta^a{}_\mu, \delta^a_b] = e_a{}^\mu \partial_\rho \theta^a{}_\nu, \quad T^\rho{}_{\mu\nu}[\theta^a{}_\mu, \delta^a_b] = 2 e_c{}^\rho \partial_{[\mu}\theta^c{}_{\nu]}, \quad \omega^a{}_{b\mu} = 0.$

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Covariance and Invariance of field theories - Teleparallel Gravity

Ingredients for dynamical equations in physics:

- dynamical physical fields $\Psi : M \to E$, — E some configuration bundle, for example some tensor bundle over M
- source fields $J : M \to E^*$, — E^* the dual configuration bundle
- background structure Σ, — for example a spacetime metric and a connection

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Notation : " $\cdot$ " denotes action of f on field under considerration

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[Giulini 2006]

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Generalize to arbitrary mappings f, which act on dynamical fields, sources and backgrounds

Covariance: Are the field equations formulated geometrically - coordinate independent, basis independent, …?
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Case A: Full covariant formulation

- Ψ : tetrad and connection $\theta^a, \omega^a{}_{b\mu}$
- J : energy momentum tensor $\Theta_a{}^\mu$
- Σ : $\emptyset$

Case B: pure tetrad

- Ψ : tetrad θ^a
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Case C: background connection

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Behaviour under general local Lorentz transformations:

Case A:
- all theories are covariant and invariant

Case B:
- only TEGR is covariant and invariant

Case C:
- only TEGR is invariant
- modified teleparalle gravity theories are covariant

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- Σ : $\omega^a{}_{b\mu} = 0$

Behaviour under general local Lorentz transformations:

Case A:
- all theories are covariant and invariant

Case B:
- only TEGR is covariant and invariant

Case C:
- only TEGR is invariant
- modified teleparalle gravity theories are covariant

1. Teleparallel/Weitzenböck Geometry

2. Symmetry and Covariance in teleparallel theories of Gravity

3. **Conclusion**

Conclusion

Geometry of manifolds in terms of tetrads and connections implies transformation behaviour of spin connection
No physical field theory involved.

Generalize to arbitrary mappings f, which act on dynamical fields, sources and backgrounds

Covariance: Are the field equations formulated geometrically - coordinate independent, basis independent, …?
Invariance: Is there a non-trivial mapping on the dynamical fields which maps solutions into solutions?

Fully covariant formulation of teleparallel theories of gravity is invariant under local Lorentz transformations.

Let us be clear with the notions of covariance and invariance in tele parallel Gravity

A geometric formulation of the geometry of spacetime requires covariant formulation of tele parallel geometry.

TEGR dynamics are special, since they are independent of the spin connection.